COMPRESSED IMAGE PRODUCTION, STORAGE, TRANSMISSION AND PROCESSING

BACKGROUND OF THE INVENTION

This invention relates to a method for producing an image of an object storing, transmitting and processing the same.

In this application, "object" means any entity that can be defined, in principle, by geometrical and/or mathematical data and/or geometrical or mathematical or empirical relationships, such as functions, correlations, regressions, lines and surfaces, etc. It is irrelevant whether the object is so complex that the number of data and/or relationships required to define it is so great that complete or exact definition is practically impossible. It is also irrelevant how many dimensions the object has. The object may be a physical one, such as a picture, a line, a surface, a solid, a tri-dimensional object or a landscape, etc., or an abstract one, such as a tensor, a form defined in a continuum having more than three dimensions, etc.; or it may be constituted by an array of data which have only a conceptual relationship with one another.

"Image" means any entity that represents an object exactly, or more or less approximately. The image may have the same nature as the object it represents, as when, e.g., it is the reproduction of a picture or an array of data representing another array of data; it may be an image in the common

meaning of the word, as when, e.g., it is a picture of a person or a landscape; or it may be quite different in nature from the object, as when, e.g., it consists of a plurality of numerical data representing a physical entity. "Intermediate image" means an image that is produced for the purpose of transforming it later into a different image of the same object, as when, e.g., a set of numbers temporarily represent a geometrical form and a geometrical image is to be developed from them. When such transformation occurs, the image finally produced will be called hereinafter "the final image". An image which is to be processed in any way elaborated to produce another image of the same nature - e.g. a first set of numbers from which another set of numbers is to be obtained, by any appropriate procedure, said other set of numbers being an intermediate or a final image, will be called a "temporary image", which, if the processing is a correction or adjustment, is an "unadjusted image".

In a great many technical processes, an image of an object must be produced, and quite often must be stored, transmitted or processed. For instance, it is a common occurrence that two-dimensional figures or pictures be represented by digital data which are stored, processed and transmitted, according to needs. This occurs in word processing by computers, message transmission by telefax, etc. Three-dimensional objects, including landscapes, may be represented by a process that is essentially the same. The representation of objects which have more than three dimensions involves in principle no conceptual departure from the said methods. Another common occurrence is the representation, storage and processing of data representing physical relationships, statistical

regressions or ways of experimental data. The use of mathematical models is also an instance of object representation by an image, which may be constituted by an array of digital data.

It is obviously desirable to reduce as much as possible the amount of data defining the image which represents a given object, without disorting the image to the extent that it might cease to represent the corresponding object with an acceptable degree of accuracy. Such a reduction of the required data, or "data compression" or "image compression", as it is sometimes called, serves to simplify, reduce and render more economical the equipment required for the storage of an image, its processing and transmission. For instance, it is well known that in modern technology, transmission lines, including frequency bands available for radio transmission, are increasingly overcrowded, and every effort is being made to exploit them as fully as possible, one of the means for so exploiting them being to reduce the amount of data that are sent through a given transmission line in order to convey a given amount of information.

It is a general purpose of this invention to provide a method for producing the image of an object of any kind, storing it, processing and transmitting it, while minimizing the amount of data that are required for carrying out the said operations.

More specific objects of the invention and specific applications thereof, will become apparent as the description proceeds.

BRIEF SUMMARY OF THE INVENTION

The following considerations are preliminary to an understanding of the process according to the invention. If the object is defined geometrically or analytically - whether by a graphic representation or a model, depending on the nature of the object, or by an array of numerical data which are assumed to define the object or in any suitable way - it may be broken u- into, viz., be considered as defined by, a plurality of components, such as lines or surfaces defined in a space which may have more than three dimensions, arrays of numerical values or functions or operators which can be represented by such lines or surfaces. For the sake of simplicity, the process according to the invention will be described firstly with a reference to an object which may be broken up into a number of plane lines, corresponding to functions of one variable. Description and definition of the process will be then expanded to those objects which must be broken up into surfaces in a three-dimensional space or in hyperspace, having more than three dimensions, corresponding to functions of two or more than tw variables. Essentially the process, as described and defined, extends to compressed images of any objects that can be defined by an array of data, by software or hardware for the production and/or elaboration of digital values, such as a special purpose computer or a computer program, or by an analogical circuit or special purpose analogical computer or analogical computer program, or by digital or analogical sensors, or the like. In what follows, the term "object" will be construed as preferably meaning the physical entities and/or relationships by which the object is defined or into which the object has been translated, and which will have been stored or

memorized, as in an electronic memory, e.g. in the form of digital values or instructions relative thereto or analogical representations of functions or relationships.

In one of its simplest forms, the object, an image of which is to be constructed, may be a plane line. The object line, as any other object, may be defined in many different ways, but, for the purposes of illustration only, it will be treated as defined by a graph or by a corresponding function, it being evident that the information conveyed by a graph can be conveyed in other suitable way. In any case, in order to carry out the process according to the invention, the object line must be translated into digital values or into a computer program or subroutine or an analogical process or into the structure of a special purpose digital or analogical computer, which can be entered and memorized in an elaborator, and which define couples of values x, y for each point of the line. The object line may be considered in its entirety, or, more frequently, it will be divided into segments, to each of which the process of the invention will be separately applied. Therefore, if the line has been so divided, the expression "object line", when used hereinafter, must be construed as meaning the particular segment under consideration at the moment .

The process, then, comprises, in a restrictive definition, the following steps:

(1) Approximating a line by a model which includes at least one differentiable component.

(2) Establishing the maximum allowable error ε and the degree k of the Taylor polynomials by which the differentiable component(s) of the model are to be approximated.

(3) Establishing at least a pitch grid h and constructing a grid each region of which has one of said pitches h.

(4) Computing the coefficients of the Taylor polynomials of the aforesaid differentiable component or components at selected points of said grid.

Two or more of the aforesaid steps may be carried out concurrently, in whole or in part, or divided into successive stages, which may be intercalated to a greater or a lesser extent.

Further operations, hereinafter described, may be carried out and are often desirable to minimize the effect of inaccuracies in the said coefficients, for rounding them off, for taking into account different scales which may be present in the data, and for obtaining, if desired, an image which has the same nature as the object. "Non-differentiable component" means herein a component comprising one or more points at which it is not differentiable, or, a component that is not differentiable at all its points.

The process according to the invention can be extended to objects that are more complex than plane lines by simple generalizations, as will be explained hereinafter.

BRIEF DESCRIPTION OF THE DRAWINGS

The invention will be better understood from the following description of preferred embodiments, with reference to the appended drawings, where:

Figs. 1a 1nd 1b illustrate an example of an object line and its image, respectively;

Figs. 2a and 2b illustrate a temporary image line the segments of which d not match at meeting points, and a corresponding adjusted image line, respctively;

Figs. 3a, 3b, 3c, and 3d illustrate respectively an object line and the corresponding model line, final image and non-differentiable component of the model, with reference to Example 1;

Figs. 4a and 4b represent a picture and its image, respectively, with reference to Example 2;

Fig. 5 represents a processed image of the picture of Fig. 4a, with reference to Example 3; and

Figs. 6a and 6b represent the negative of the picture of Fig. 4a and its image, respectively, with reference to Example 4.

DETAILED DESCRIPTION OF PREFERRED EMBODIMENTS

The process steps hereinbefore defined will now be more fully explained.

Step (1) - The object line, the data defining which have been physically stored e.g. in an electronic memory, is approximated by a model, preferably defined in the same way as the object line, which model preferably consists of at least a first component embodying the characteristics of the object, if any, which render it non-differentiable at some points or regions - it being of course possible to omit said first component if there are no significant characteristics of non-differentiability of the object - and at least a second component which embodies all the differentiable content of the object. Typical cases of models are the following:

Case a) The first component is a base line, which is a simple - desirably, the simplest - line having qualitatively the same discontinuities as the object line, and the second component is a curve which represents the deviations therefrom of the object line, and which will be differentiable and can be called interpolating function. The base line may be constructed in each individual instance, or, more conveniently, may be chosen, according to the actual discontinuities of the object line, from a number of normal forms, which are the simplest functions having the required discontinuities. The following standard form of model can be used in this case:

(1) $\Phi(x) = Hx_o,a,b,c,d(x) + \phi(x)$

wherein H is a normal form defined by $H(x) = a(x-x_o) + b$, if $x \geq x_o$ or $H(x) = c(x-x_o) + d$, if x is less than x_o. The values of the parameters x_o,a,b,c,d are determined, in a preferred embodiment of the invention, by minimizing a quantity representing an error, e.g. the quadratical error, as hereinafter set forth. The base line can be predetermined, or chosen, in general

according to predermined criteria, from a list prepared in advance, or it can be chosen in each case by the operator. This case is illustrated at Fig 1a, 1b showing respectively an object line and its model.

Case b) The model is a differentiable function of another function which embodies the non-differentiable characteristics, viz the discontinuities, of the object line. It can be epressed as:

(2) $\Phi(x) = \Phi'[\phi(x)]$,

wherein ϕ is the first component , which will be called the base curve, and Φ' is the second component. $\phi(x)$ can be looked at as defining a change of coordinates: in the differentiable component Φ', the ordinates are referred to abscissae which are not x, but $\phi(x)$.

Case c) This case will be mentioned here, though it is not applicable to a line, but only to surfaces in a space having three or more dimensions. In the case of three dimensions, a coordinate (say, the elevation) z of a surface, is a function z_1 in a certain region of the plane x-y of the two remaining coordinates and is another function z_2 in another region thereof, the two regions being separated by a border line defined e.g. by a relationship $y=\phi(x)$. Then the model $\Phi(z,y)$ consists of the function z_1 if y is greater than $\phi(x)$, and z_2 if y is smaller than $\phi(x)$, one or the other of the z_1 and z_2 applying when $y=\phi(x)$.

Case d) The object line is differentiable at all points, and the model consists only of a differentiable component.

In a form of the invention, all the parameters of the model the values of which have to be chosen, are determined by minimizing a quantity representing an error - e.g. the quadratical error, viz. $\Sigma[f(x_i) - \Phi(x_i)]^2$ - the

minimization being carried out by means of a predetermined subroutine with respect to all the parameters of the model Φ, for the function f(x) representing the object, the values of f(x) for each x being determined by known subroutines. Programs for this purpose are available, e.g. from the ILSM library.

Step (2) - a) - The maximum allowable error ε, which is to be tolerated i approximating the object line, viz. which expresses the desired precision of the image, is established.

b) - The degree k of the Taylor polynomials, which will be used to approximate the differentiable component or interpolating curve, is established.

Step (3) - The grid need not be cartesian and its coordinate lines may be curved, although for simplicity's sake a cartesian grid will always be illustrated herein. The grid may be divided into different regions having different grid pitches or even different types of coordinate lines. The grid pitch h (viz., the distance between adjacent coordinate lines which define the grid cells) is selected according to the precision desired of the image, and may be different in different parts of the region, although a regular grid will often be preferred.

In an embodiment of the invention, h is calculated, by a suitable subroutine, from the formula

(3) $CMh^{k+1} \leq \varepsilon$

wherein $C = 1/(k+1)!$ and M is the maximum, at each grid point, of the absolute value of the (partial, in the case of an object which is a function of more than one variable) derivatives of degree k+1 of the differentiable component or components, in the segment or zone of the object under consideration, M being determined by using a known subroutine which computes the derivatives of order k+1, produced e.g. by a package such as MAXIMA OR MATHEMATICA.

Step (4) - The nodes of the grid are taken as base points, and a (known, e.g. a MAXIMA) subroutine is applied at each base point to compute the Taylor polynomials of degree k of the interpolating curve.

At this stage, the following data have been obtained:

A) The coefficients of the Taylor polynomials of the differentiable component or components of the model;

B) The number or other identification or analytical definition of the non-differentiable component(s) , if any, of the model, such as the base line or the base curve;

C) The values of the parameters of the said non-differentiable component(s), if any;

and these define an image, which will usually be an intermediate image, but could be a final one, according to cases. Hereinafter it will be assumed that it is an intermediate image, from which the final image, in the same form as the original object, is to be constructed; however this is done merely for the sake of simplicity and involves no limitation.

In many cases, as will be explained below, the image thus obtained may require further elaboration without changing its nature, viz. while remaining a set of data of the same kind, and it will be only a temporary, in particular an unadjusted image. Then some or all of the steps from (5) on will be carried out.

Step (5) - In the case of the presence of so-called noise or inaccuracies i- said temporary image line, or if the numerical noise, viz. the inaccuracies of the computations, which are large in comparison with the accuracy required, the Taylor polynomials which make up the temporary image line or its differentiable component may disagree at their meeting points by more than allowed by the required accuracy, as represented, by way of example, in Fig. 2a.

In this case, an adjusted image line is constructed by applying to each differentiable component a subroutine, hereinafter "Whitney subroutine", which computes W, wherein W is a quantity representing the discrepancies of the Taylor polynomials. In particular, W can be given by t formula:

(3) $W = \Sigma_{i,j} \| p_i - (p_j)_i \|^2$

Here the sum is taken over all the adjacent grid points i, j (possibly belonging to different segments of the image). p_i, p_j denote the Taylor polynomials, obtained in steps (1) - (4) at the grid points i, j, and $(p_j)_i$ denotes the polynomial p_j, expressed in coordinates, centered at the i-th grid point. $\| p - q \|^2$ denotes, for any two polynomials p and q of the same degree and number of variables, the sum of squares of the differences of corresponding coefficients.

For any values of the coefficients of p_i, W is computed by using known subroutines, produced e.g. by a package such as MATHEMATICA.
W is then minimized (e .g. by standard gradient methods), using, as starting values of the coefficient of the Taylor polynomials, those obtained by the previous steps, and under such constraints that the result of the minimization do not deviate from the initial data by more than the allowed error, e.g. under the condition that the zero degree coefficients of sai polynomials remain unchanged. An adjusted image line, corresponding to the unadjusted image line of Fig. 2a, is illustrated by way of example in Fig. 2b.

Step (6) - If the accuracy of the adjusted coefficients of the Taylor polynomials obtained from step (5) is excessive with respect to that desired in the final image, they are rounded off to a maximum allowable error ε' by any suitable method (not necessarily the same for coefficients of different degrees). The data thus obtained represent the adjusted image line.

Step (7) - Sometimes the data of the object to be represented may require the use of different grid resolutions, or such use may be desirable. An example which clarifies this case is the following.
Let us assume that the object represents a periodic phenomenon, e.g. an oscillatory phenomenon such as an oscillating eletcrical impulse or an electromagentic wave. Such a phenomenon can be analyzed and is usually represented by the combination of two or more superimposed components, specifically, a relatively low frequency carrier wave and a higher frequency modulating wave. The modulation can be sometimes considered as

resulting from a first, intermediate frequency modulation, and one or more high frequency modulation or modulations, and in this case the object will have three or more components. The image can be conveniently constructed from images of ther various components, e.g. of the carrier wave and of the modulating wave or waves, and obviously the lower frequencies will require lower resolutions and larger grid pitches will be suitable for them. Likewise, the frequency of an oscillatory phenomenon may vary at different times or in different spatial regions and its components will not be superimposed, but separated in space. Similar situations may occur in various cases. Generally, many kinds of object may comprise superimposed or separated components which have details of different fineness, which require different degrees of resolution. Since oscillatory phenomena are a typical case of objects requiring different grid resolutions, the word "frequency" will be used to indicate the fineness of the required grid, but this is not to be taken as a limitation, since the same procedure can be applied to non-oscillatory phenomena.

In such cases, the following procedure is preferably followed:

a) Steps 1 to 6 (or such among them which are necessary in the specific case) are carried out and a first temporary image is obtained.

b) A new maximum error ε_2, bigger than ε (or ε', as the case may be) is chosen.

c) A grid which is sparser than the one used for carrying out the steps under a), and the pitch of which is determined by the resolution required by the lowest frequency of the components existing in the object (e.g. that of a carrier wave) is established.

d) Steps 1 to 6 are repeated using ε_2 and the sparser grid and a second temporary image is obtained.

e) The second temporary image thus obtained is substracted from the first and a first residual image is obtained, which contains only data relating to higher frequency components of the object.

f) The same procedure - steps b) to e) - is repeated for successively higher frequencies of components, correspondingly obtaining successiv- residual images increasingly restricted to higher frequency components.

As a result, coefficients of Taylor polynomials are obtained on several grids having increasingly higher resolutions, viz. smaller pitches, separately corresponding to the object components requiring increasingly higher resolutions.

The data obtained after steps (1) to (4) and those among (5), (6), (7), which it has been found necessary to perform, constitute an intermediate image or someyimes a final one. Usually these are the compressed data which can be stored, transmitted and processed.

If a further compression is desirable, one of the standard methods of encoding coefficients (e.g. Hoffman coding) can be applied. If necessary, the resulting string of data can be further compressed by one of the standard methods of unstructured data compression (e.g. entropy compression). However, this last step reduces the possibility of a compressed data processing.

If a final image, which has the same nature as the object, is to be constructed, the following procedure is followed:

Step (8) - a) The Taylor polynomial coefficients obtained after completion of steps (1) to (4) and of those among steps (5) and (6) which it has been found necessary to perform, are treated as if they represented an unadjusted temporary image, which is affected by noise, and are subjected once more to step (5), using them as starting data.

b) The domain in which the temporary image has been defined is divided into regions by means of a grid, each region being a portion of the grid around a grid node or base point. These regions may overlap.

c) A curve or curves representing the Taylor polynomials of degree k in the above regions are constructed from the coefficients defining the temporary image - e.g. obtained as in step (8) a) - at each node of the grid or of that grid having the highest resolution (smallest pitch), if there are more than one grid (particularly if step (7) has been carried out), using a known subroutine.

Said curve or curves constitute the final image of the object line.

The aforementioned curves may diverge at the meeting points of the regions mentioned above under b) (or on their overlapping parts). If this disagreement does not exceed the allowable error ε, any of the overlapping curves curves can be used at the meeting points on the overlapping parts of the above regions.

If as the result of the noise of the data or the computational noise, the above discrepancies are large in comparison with the accuracy required, average values can be used on the overlapping parts. This is done by

averaging the values of the overlapping curves with the appropriate weights.

Actually, other polynomials or functions could be used for approximation purposes, such as Tchebicheff polynomials, trigonometric exponential functions, etc., without departing from the invention, but Taylor polynomials are preferred.

The above described process applies, with obvious generalization, to a wide range of objects. Some examples follow.

I - A surface in a three-dimensional space corresponds to a function of two variables. If the surface is defined in a space that has more than three, say, n+1 dimensions, the independent variables will be more than two, say, n (x_1,x_2,...x_n), but the operations to be carried out will be essentially the same, and the necessary generalizations will be obvious to skilled persons. In any case, any surface can be translated, as well as a line, to digital values, which can be entered and stored. The model will be constructed in the same way as for a line. Case c) of model construction, already described, applies to surfaces in any space. Analogously to case a), a model may consists of a simple base surface, which presents the discontinuities of the object surface, and by an differentiable or interpolating surface, which represents the deviations of the object from the base surface. One can also operate analogously to case b), by using functions of more than one variable. The minimization of the quadratical error is effected in the same way as in the case of an object line, using values of $\Phi_{ij...n}$ and $f(x_i,x_j,....,x_n)$

which depend on n variables. The remaining steps are likewise adapted to the existence of n variables. All derivatives, of course, will be partial derivatives. The construction of the final image from the temporary image - step (8) - can likewise be carried out with obvious generalizations in the case of images defined in a space having any number of dimensions.

II - A surface can be considered as a family of lines, which are obtained by the intersection of the surface with a family of planes, e.g. vertical planes the orientation of which is taken as that of the x-axis, identified by a parameter, e.g. their y coordinate. A family of curves in a plane, depending on one parameter, as may result from the representation of any number of phenomena, is obviously equivalent to that of a surface and may be treated as such, or vice versa.

III - A particular case of an object which is a surface is, e.g., a terrain, wherein the surface is defined by the elevation as a function of two plane (cartesian or polar) coordinates.

IV - A building can be represented in the same way, if it is very simple. If its shape is complex, however, it must be broken up into a number of component parts. However, if it is desired to represent it as it is seen from the outside, say by an observer which can place himself at any vantage point within a certain distance from the building, the oberver's position can be identified by three coodinates, x, y and z (or polar coordinates), or by two, if it assumed that the observer's eye is at a given level. From each position of the observer point it is possible, if the configuration of the terrain is

known, to determine the distance D on each line of sight from the observer's eye to the building surface, and this will determine how the building is seen. Each line of sight can be identified by two coordinates: e.g. its inclination (the angle thereof with the vertical in a vertical plane which contains it), and its azimuth (the angle of said vertical plane with a reference vertical plane, e.g. one that contains the geographic or magnetic north). The way in which the building appears to the observer, is therefore defined by a function D of five variables, viz. by a surface in a six-dimensional space.

V - A family of curves in a plane, depending on more than one parameter, is obviously equivalent to a surface in a space having more than three dimensions.

VI- If in example IV above the coordinates of the observer are known as a function of a single variable, say, when he approaches the building along a given line, in which case the variable is the distance covered from a starting point, or in motion, as in a vehicle, along a given line, in which case the variable is time. In this case the variables of the surface become three (e.g. distance or time and inclination and azimuth) and the space is only four-dimensional, but the four-dimensional surface is subject to the constraint represented by the definition of the observer's motion. In general, in many cases, the degree of the space in which the surface is defined may be reduced by the introduction of suitable constraints.

VII - The final image of a colour picture is another colour picture, that is not identical, but sufficiently similar to the object picture. The object picture can be scanned by known apparatus (scanners), by means of white light, and for each point the intensity of the three basic colours (magenta, cyan and yellow) may be measured and registered. The object is thus reduced to three partial or component objects, each consisting of the distribution of one basic colour over the picture and having a physical reality, as it i equivalent to the colour picture that would be contained by exposing the original through three filters, having colours complementary to the three basic colours, or, in practice, to an array of digital data representing such one-coloured picture. Each of said partial objects can be subjected to the process of the invention, to produce a reduced or compressed array of data, constituting a partial image, and the partial images can be transformed into a combined final image approximating the original object, by processes known to those skilled in the art. If the partial images must be stored and/or transmitted, the process of the invention will facilitate doing this and render it more economical. In the same way a dynamic coloured picture, such as a movie or a TV broadcast, can be reduced to a final dynamic image.

A particular advantage and a preferred aspect of the invention consists in the possibility of processing the compressed intermediate image obtained as set forth hereinbefore and producing from it a processed final image, which does not represent the object but represents what would have been the result of processing the object. The processed intermediate image can be stored and transmitted with the already mentioned savings and

advantages inherent in the reduction of the number of data, but said reduction is even more advantageous in the processing, for it is obviously more convenient to process a reduced instead of a larger amount of data. Said processing in a compressed form, as it may be called, is made possible by the following property:

Let F be an operator which is analytic in nature, viz. can be defined by mathematical relationships. Let O be an object of any nature, but which can be represented by Taylor polynomials p_i. Then by applying operator F to the p_i's, one obtains polynomials which represent the object that would be obtained by applying the operator F to the object O. If one uses the symbol $\approx$ to indicate that an array of polynomials represents an object, one can write:

$$\text{if} \quad p_i \approx O, \quad \text{then} \quad F(p_i) \approx F(O).$$

Elementary examples of analytic operators are algebraic operations, rotations of geometrical figures, changes of coordinates in general, etc. These operators are represented by mathematical operations. If F(O) is to be constructed, such operations must be carried out on all the data, e.g. digital data, which define the object. But if a compressed image has been obtained as set forth above, and an array of Taylor polynomial coefficients has been obtained, which are in a much smaller number than the said digital data, said mathematical operations can be carried out on said coefficients, and a processed intermediate image will be obtained, which represents F(O) and from which F(O) can be constructed as set forth in step (9) above.

The following examples illustrate a number of embodiments of the invention.

Example 1

An object line f in the plane (x, y) is given by an array

$A = (y_o, y_1, \ldots, y_{100})$, where $y_i = f(x_i)$, $x_i = i/100$, $i = 0, 1, \ldots, 100$. In this specific example the array (array 1) is the following:

0.1152	0.1155	0.1191	0.1131	0.1174	0.1133	0.1108	0.1149	0.1105
0.1182	0.1167	0.1206	0.1238	0.1196	0.1264	0.1282	0.1313	0.1315
0.1299	0.1330	0.1366	0.1409	0.1402	0.1462	0.1569	0.1608	0.1631
0.1604	0.1693	0.1779	0.1797	0.1826	0.1826	0.1888	0.1963	0.2011
0.2034	0.2084	0.2170	0.2244	0.2265	0.2327	0.2429	0.2468	0.2472
0.2523	0.2661	0.2673	0.2702	0.2796	0.2811	0.2845	0.2949	0.3022
0.3078	0.3049	0.3121	0.3157	0.3256	0.3270	0.3346	0.3413	0.3405
0.3428	0.3503	0.3515	0.3530	0.3571	0.3675	0.3616	0.4648	0.4665
0.4659	0.4607	0.4600	0.4536	0.4473	0.4441	0.4427	0.4330	0.4329
0.4268	0.4243	0.4185	0.4135	0.4107	0.3961	0.3925	0.3877	0.3774
0.3698	0.3671	0.3583	0.3449	0.3397	0.3338	0.3271	0.3091	0.3031
0.2929								

An object line itself is shown in Fig. 3a. The required accuracy of representing this line is 0.035. The compressed image of this line is produced as follows.

Firstly it is subdivided into three segments lying over the segments [0.0, 0.6], [0.6, 0.8], [0.8, 1.0] in the x-axis. The following model is chosen on the segments [0.0, 0.6] and [0.8, 1.0]:

$y = Q(x) = c_1 \sin(\omega_1 x + \phi_1) + c_2 \cos(\omega_2 x + \phi_2) + c_3 x^2 + c_4 x + c_5$ with c_1, c_2, ω_1, ω_2, ϕ_1, ϕ_2, c_3, c_4, c_5 - the parameters.

On the segment [0.6, 0.8] the following model is chosen:

$y = Q(x) + H_{x_o, a, b, c, d}(x)$, where $Q(x)$ is as above, and the normal form H is defined by $H(x) = a(x - x_o) + b$, if $x \geq x_o$ or $H(x) = c(x - x_o) + d$, if x is less than x_o. Said normal form is illustrated in Fig. 3d. Approximation on each segment is carried out by minimization, with respect to the corresponding parameters, of the quadratic error:

$\Sigma (y_i - Q(x_i))^2$ ($\Sigma (y_i - Q(x_i) - H(x_i))^2$ on [0.6, 0.8]).

The values of the parameters found are given in the following array 2.

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Q(x) = 2.0  + 0.1*x - 0.2*x*x - 0.15*cos( -0.4+4*x) -
       0.2*sin( -0.3 + 0.5*x )

H(x) =  1.0/7.0 * (x-0.7) + 0.1  ,  x < 0.7
H(x) = -1.0/3.0 * (x-0.7) + 0.2  ,  x >= 0.7
```

The corresponding model curve is shown in Fig. 3b.

The error of the approximation of the object line by the model found turns out to be 0.005. Respectively, on the step 2, ε is chosen to be 0.03. k is chosen to be 2 on each segment.

M, equal to the maximal absolute value of the third derivative of the smooth component in the above model, as computed by the standard subroutine, 8. The maximal possible pitch h of the grid to be constructed, is defined by $(1/6)\ M\ (h/2)^3 = \varepsilon$, or $h \approx 0.24$. In order to provide a uniform grid, a smaller value $h = 0.2$ is chosen on each segment. The corresponding grid points are the following: 0.1, 0.3, 0.5 on [0.0, 0.6], the only grid point 0.7 on [0.6, 0.8] and the only grid point 0.9 on [0.8, 1.0]. Taylor polynomials at these points, as computed by the standard "MATHEMATICA" subroutine, are given in the following array 3.

Zi	a0	a1	a2
0.1	0.121582031250	0.105957031250	0.993652343750
0.3	0.180175781250	0.454345703125	0.632080078125
0.5	0.285644531250	0.542724609375	-0.236083984375
0.9	0.381103515625	-0.727050781250	-1.394042968750
0.7	0.272460937500	0.125244140625	-1.083496093750

a= 0.142822265625 b= 0.099853515625

c= -0.333251953125 d= 0.199951171875

Now the coefficients of order 0 are rounded off up to 3 digits, the coefficients of order 1 are rounded off up to 2 digits and the coefficients of order 2 up to 1 digit. The parameters of the normal form H are rounded off up to three digits. These data, listed in the following array 4 represent the intermediate compressed image.

Zi	a0	a1	a2
0.1	0.121	0.10	0.9
0.3	0.180	0.45	0.6
0.5	0.285	0.54	-0.2
0.9	0.381	-0.72	-1.3
0.7	0.272	0.12	-1.0

a= 0.142 b= 0.100

c= -0.333 d= 0.200

The compression ratio is 4*100 digits/ 37 digits ≈ 10.8.

The final image is obtained by computing the values of the Taylor polynomials (and the normal form H on [0.6, 0.8]) at the initial points x_i, i = 0, ..., 100. Each polynomial is used for x_i belonging to the corresponding cell of the grid z_i. The result is shown in the following array 5.

0.1196	0.1190	0.1185	0.1183	0.1182	0.1183	0.1186	0.1190	0.1197
0.1205	0.1215	0.1227	0.1240	0.1256	0.1273	0.1292	0.1313	0.1335
0.1360	0.1386	0.1426	0.1460	0.1496	0.1532	0.1570	0.1609	0.1649
0.1691	0.1733	0.1777	0.1822	0.1868	0.1916	0.1964	0.2014	0.2065
0.2117	0.2171	0.2225	0.2281	0.2318	0.2376	0.2433	0.2490	0.2546
0.2602	0.2658	0.2713	0.2768	0.2822	0.2876	0.2930	0.2983	0.3036
0.3088	0.3140	0.3192	0.3243	0.3294	0.3344	0.3380	0.3424	0.3466
0.3506	0.3545	0.3581	0.3615	0.3648	0.3678	0.3706	0.4709	0.4685
0.4660	0.4633	0.4603	0.4572	0.4539	0.4503	0.4466	0.4427	0.4376
0.4328	0.4276	0.4223	0.4166	0.4107	0.4046	0.3981	0.3915	0.3845
0.3773	0.3699	0.3621	0.3542	0.3459	0.3374	0.3287	0.3196	0.3104
0.3008								

The corresponding final curve is shown in Fig. 3c. The maximal error in representing the object curve by the final one is 0.033.

Example 2

The object (black and white, continuous tone) picture is the standard test picture, called "Lena" (see Fig. 4a). It is represented by a 512 x 512 array, each pixel containing 8 bits, representing one of the gray levels between 0 and 255. The file representing this picture is available in test collections in the field of imaging . A part of this array, representing the piece S, marked on Fig. 4a, is the following.

97 97 97 97 97 97 100 98 96 94 92 90 89 86 84 84 86
89 93 96 101 109 120 133 152 171 188 202 213 222 214 200 185 167
148 127 138 136 133 130 128 125 115 114 113 111 110 109

97 97 97 97 97 97 99 97 95 93 91 89 89 85 83 83 85
89 92 95 100 108 119 132 150 169 186 200 211 220 217 203 187 169
149 127 138 135 132 130 127 124 115 114 113 111 110 109

97 97 97 97 97 97 98 96 94 92 90 88 88 84 83 83 84
88 92 94 100 108 118 132 148 167 184 198 209 218 221 206 189 171
150 128 137 134 132 129 126 124 115 114 113 111 110 109

97 97 97 97 97 97 98 96 94 92 90 88 87 84 82 82 84
87 91 94 99 107 118 131 146 165 182 196 207 216 224 208 191 172
152 129 136 134 131 128 126 123 115 114 113 111 110 109

97 97 97 97 97 97 97 95 93 91 89 87 87 83 81 81 83
87 90 93 98 106 117 130 144 163 180 194 205 214 227 211 193 174
153 130 136 133 130 128 125 122 115 114 113 111 110 109

97 97 97 97 97 97 96 94 92 90 88 86 86 82 81 81 82
86 90 92 98 106 116 130 142 161 178 192 203 212 230 214 196 176
154 131 135 132 130 127 124 122 115 114 113 111 110 109

99 96 94 93 93 94 93 94 93 92 90 87 85 85 85 85 85
85 86 90 96 104 114 125 136 145 158 175 195 219 220 222 213 193
161 117 134 132 129 127 125 122 121 118 115 111 108 105

95 93 91 91 91 92 95 95 95 93 91 88 84 84 84 84 84
84 87 91 97 104 113 124 134 144 157 174 194 218 220 223 215 195
163 120 133 131 129 127 125 123 121 118 115 111 108 105

92 90 89 88 89 91 96 96 96 95 92 89 84 84 84 84 84
84 88 92 97 104 112 123 133 143 156 172 193 216 220 224 216 196
165 122 131 129 128 126 125 123 121 118 115 111 108 105

88 86 86 86 87 89 97 98 97 96 94 91 84 84 84 84 84
84 89 92 97 103 112 122 132 141 154 171 191 215 220 224 217 198
167 125 128 127 125 124 123 122 121 118 115 111 108 105

84 83 83 83 85 87 99 99 99 97 95 92 86 86 86 86 86
86 91 93 97 103 111 121 130 140 153 170 190 214 221 225 218 199
169 127 124 123 122 122 121 120 121 118 115 111 108 105

80 80 80 81 83 86 100 100 100 99 96 93 88 88 88 88 88
88 92 94 97 103 110 119 129 139 152 168 189 212 221 226 219 201
171 130 119 119 118 118 118 118 121 118 115 111 108 105

65	71	78	85	91	98	100	100	100	101	101	102	102	100	97	94	89
83	91	91	93	97	105	115	128	143	157	172	187	201	223	229	223	205
174	130	118	118	118	118	118	118	118	114	111	107	103	99			

65	72	79	85	92	99	104	104	104	104	104	104	107	105	103	99	95
90	89	88	90	95	102	112	128	143	158	172	187	202	219	226	220	202
172	129	118	118	118	118	118	118	115	112	109	105	102	99			

66	73	79	86	93	99	106	105	105	104	104	104	108	107	105	102	98
94	86	85	87	92	99	109	127	141	156	171	185	200	214	221	216	198
168	126	118	118	118	118	118	118	113	110	108	105	102	99			

67	73	80	87	93	100	105	104	103	102	101	100	107	106	105	102	99
95	83	83	85	89	97	107	123	138	153	167	182	197	207	214	210	193
163	121	118	118	118	118	118	118	112	110	107	105	102	100			

67	74	81	87	94	101	101	100	99	97	96	95	103	103	102	100	97
93	81	80	82	87	94	104	118	133	148	162	177	192	198	206	202	185
156	114	118	118	118	118	118	118	112	110	108	106	104	102			

68	75	81	88	95	101	95	93	92	90	88	86	97	97	96	95	92
89	78	77	79	84	91	101	111	126	141	155	170	185	187	196	192	176
147	106	118	118	118	118	118	118	113	111	110	108	106	105			

79	83	88	93	97	102	94	69	48	30	15	4	40	51	59	65	68
68	80	80	82	85	88	93	94	110	128	146	164	184	192	176	160	146
132	120	116	115	115	115	115	114	107	106	104	102	98	94			

80	83	86	90	93	96	89	68	50	36	26	19	44	53	59	62	63
61	71	72	74	76	80	84	86	102	119	137	156	176	185	171	157	145
133	122	117	116	115	115	114	113	109	108	106	104	100	96			

81	83	85	87	89	91	86	69	56	46	39	36	53	59	63	64	63
58	66	67	69	72	75	80	81	97	114	132	151	171	179	166	155	144
134	125	118	117	116	114	113	112	110	109	107	104	101	96			

84	85	85	86	87	87	86	73	64	58	55	56	66	70	71	70	67
60	65	66	68	70	74	78	80	96	113	131	150	170	173	162	152	143
134	127	119	117	116	114	113	111	110	109	107	104	101	96			

88	87	86	86	85	84	89	80	74	72	74	79	83	85	84	81	75
66	67	68	70	72	76	80	82	98	115	133	152	172	166	157	149	141
135	129	120	118	116	114	112	110	109	108	106	104	100	96			

92	90	88	86	84	82	94	89	88	90	95	104	105	105	102	96	88
77	73	74	76	78	82	86	88	104	121	139	158	178	160	152	146	140
136	132	121	119	116	114	111	109	107	106	104	102	98	94			

```
107 104 101  98  94  91  86  84  83  83  84  86  91  97  98  93  83
 68  62  71  82  96 113 132 158 162 166 168 170 171 154 148 141 134
128 121 124 122 119 116 114 111 105 104 102 101 100  98

113 110 107 105 102  99  91  89  88  88  89  91  94 103 106 104  97
 84  87  96 107 121 138 157 169 172 174 176 176 176 154 147 140 134
127 120 124 121 118 116 113 110 104 103 102 100  99  98

117 114 112 110 108 106  95  94  93  93  94  95  99 111 117 117 113
103 105 114 125 139 156 175 178 179 180 180 179 178 153 146 140 133
126 120 123 120 118 115 112 110 104 102 101 100  98  97

119 117 115 114 112 110 100  98  97  97  98 100 106 120 129 132 130
123 116 124 136 150 166 186 184 184 184 182 180 177 152 146 139 132
126 119 122 120 117 114 112 109 103 102 100  99  98  96

119 118 117 115 114 113 105 103 102 102 103 105 114 131 142 148 149
144 119 128 139 153 170 189 187 186 184 182 178 174 152 145 138 132
125 118 122 119 116 114 111 108 102 101 100  98  97  96

118 117 116 115 114 114 109 108 107 107 108 109 125 144 158 167 170
168 116 124 136 150 166 186 188 185 182 178 173 168 151 144 138 131
124 118 121 118 116 113 110 108 102 100  99  98  96  95

118 118 118 118 118 118 111 110 111 114 118 125 131 156 173 183 184
177 152 145 145 153 169 194 185 184 182 180 179 177 154 145 136 129
123 117 112 114 114 112 108 102  96 108 111 107  95  75

118 118 118 118 118 118 114 113 114 117 121 127 136 161 178 186 187
180 157 148 147 155 170 193 188 186 184 182 180 178 153 144 136 128
122 116 112 114 114 112 108 102  98 107 108 100  85  62

118 118 118 118 118 118 117 116 117 119 124 130 141 166 182 190 190
183 161 151 150 156 171 193 190 188 185 183 180 178 152 143 135 128
121 116 112 114 114 112 108 102 101 106 104  93  75  49

118 118 118 118 118 118 119 118 119 122 126 133 147 170 186 194 194
185 165 155 152 158 171 193 191 188 185 182 180 177 152 143 134 127
121 115 112 114 114 112 108 102 103 106 100  87  65  35

118 118 118 118 118 118 122 121 122 125 129 135 152 175 190 198 197
188 169 158 155 159 172 193 191 188 185 181 178 175 151 142 134 126
120 114 112 114 114 112 108 102 106 105  96  80  55  22

118 118 118 118 118 118 125 124 125 127 132 138 157 180 195 201 200
191 174 161 157 161 173 192 191 187 183 179 176 172 150 141 133 126
119 114 112 114 114 112 108 102 108 104  93  73  45   9
```

114	117	119	122	125	127	120	123	126	130	136	142	160	176	188	195	198
196	158	152	151	154	162	174	198	191	184	178	173	169	160	140	125	116
112	113	112	111	110	109	108	107	121	90	64	44	29	19			
112	115	118	120	123	126	124	126	128	132	136	141	168	184	195	202	204
202	175	168	166	168	174	185	200	194	189	185	182	180	161	140	125	115
110	111	111	110	109	109	108	107	111	82	58	39	26	18			
111	114	117	119	122	125	128	128	130	132	136	140	168	183	194	200	202
199	186	178	174	174	180	189	193	189	186	184	183	182	160	139	123	113
108	108	109	109	108	108	108	108	102	74	52	35	23	17			
111	114	117	119	122	125	130	130	130	132	134	138	158	173	183	189	190
187	190	181	176	175	179	187	179	177	175	175	175	177	158	137	121	110
104	104	107	107	108	108	108	108	92	66	46	30	21	16			
112	115	118	120	123	126	131	130	130	130	132	134	140	154	164	169	170
166	189	178	172	170	172	179	156	156	156	158	160	163	156	134	117	106
100	99	105	106	107	107	108	109	82	58	39	26	18	15			
114	117	119	122	125	127	132	130	128	128	129	130	112	126	136	140	141
137	181	169	161	158	159	164	126	127	130	133	137	142	153	130	113	101
95	94	104	105	106	107	108	109	72	50	33	22	15	14			
117	115	113	112	113	114	109	105	102	101	100	100	97	102	105	108	110
111	134	134	131	127	121	113	110	113	116	119	122	125	111	100	93	91
93	100	91	110	120	121	114	97	29	27	25	24	22	20			
100	98	96	96	96	97	95	92	90	88	88	88	87	91	95	97	99
100	117	116	113	108	102	94	81	85	89	93	97	101	91	82	78	78
82	91	100	114	120	116	104	83	25	24	23	21	20	19			
86	84	82	82	82	83	82	80	78	77	77	78	77	82	85	88	90
91	103	102	98	93	86	78	61	66	71	76	81	86	78	72	69	72
78	89	107	117	118	110	94	68	23	22	21	20	19	18			
75	73	71	70	71	72	71	68	67	67	67	68	70	75	78	81	83
83	93	91	87	82	74	65	50	56	62	68	73	79	73	68	68	73
82	95	113	119	116	103	82	52	21	20	20	20	19	19			
66	64	62	62	62	63	60	58	57	57	58	60	65	69	73	75	77
78	86	84	80	74	66	56	48	55	62	68	75	82	74	72	74	81
92	108	119	120	112	96	70	36	20	20	20	20	20	20			
60	58	56	56	56	57	50	48	48	48	50	52	61	65	69	71	73
74	83	80	76	69	61	51	55	63	70	78	85	93	83	83	87	96
110	127	124	120	108	87	57	18	20	20	21	21	22	22			

The compressed image is produced as follows: first, the picture is subdivided into square segments, c ontaining 6 x 6 = 36 pixels each one. See Fig. 4a and the array S above, where one of such segments is marked.

The step 1 consists in approximating the picture on each segment by the model, which is chosen to be the quadratic polynomial

$$z = a_o + a_1 x + a_2 y + a_{11} x^2 + 2a_{12} xy + a_{22} y^2$$

where z represents the gray level, and x and y are the coordinates on t picture plane centered at the center of the corresponding segment.

The values of the coefficient "a" are found by the standard subroutine, minimizing the quadratic error of the approximation of the gray level on each segment by the model chosen.

The array of 8 x 8 = 64 polynomials, obtained on the segments, covering the piece S of the picture, is given in the following array 7.

0.37500000	-0.0058594	-0.0055246	0.001883	-0.007146	0.010986
0.35937500	-0.0104353	-0.0276228	0.007010	0.013632	0.009208
0.31640625	-0.0081473	0.0078125	0.010149	-0.001263	0.033064
0.39843750	-0.0127790	0.0998326	0.003348	0.014263	0.062360
0.74218750	-0.0247767	0.1643973	-0.015172	0.000459	-0.049700
0.70703125	0.0304130	-0.2262835	-0.009312	-0.027809	-0.038191
0.50781250	-0.0092076	-0.0348772	-0.015067	-0.003071	0.007847
0.43359375	0.0021763	-0.0233817	0.007010	0.004908	-0.004918
0.33593750	-0.0344308	-0.0031808	0.003348	0.027637	0.026263
0.37109375	0.0164063	-0.0194196	-0.005022	0.008954	-0.022600
0.32421875	0.0092634	-0.0077009	0.016532	-0.015383	-0.003871
0.38671875	-0.0045201	0.0807478	0.006069	-0.026834	0.035575
0.63671875	-0.0199777	0.1973214	0.009626	-0.011422	0.066127
0.81250000	0.0224331	-0.2329241	0.003558	0.016875	-0.205811
0.48828125	-0.0261161	-0.0174107	-0.025321	0.024337	0.006069
0.43750000	0.0035156	-0.0440290	-0.002511	-0.010188	-0.007533
0.32031250	0.0101563	0.0811384	0.006278	0.006256	0.002511
0.40234375	-0.0292969	-0.0090960	-0.058594	-0.026260	0.015172
0.40234375	-0.0013951	-0.0376674	-0.050642	0.022643	-0.022391
0.33984375	-0.0376116	0.0617188	0.004290	-0.008265	0.046980
0.63281250	-0.0425223	0.1786830	-0.036516	0.004305	0.006173
0.80468750	-0.0712053	-0.2079241	-0.040388	0.029043	-0.234375
0.45703125	-0.0006696	0.0003348	-0.000314	-0.009184	-0.011300
0.41015625	0.0016741	-0.0319196	0.020299	0.019056	-0.000732
0.33203125	-0.0089286	0.0193080	0.020194	-0.052519	0.014544
0.21093750	0.1237165	-0.0967076	0.060059	0.156036	0.077009
0.25781250	0.0914063	0.0062500	0.087995	-0.093673	-0.055455
0.26562500	-0.0221540	0.0343192	0.074916	-0.002899	0.018101
0.47265625	-0.0229911	0.2156250	0.078055	0.008839	0.015904
0.57812500	-0.0305245	-0.1201451	-0.003557	0.072006	0.021240
0.44531250	0.0000558	-0.0220424	0.014230	-0.028096	0.001674
0.41015625	-0.0073103	-0.0318638	-0.019148	0.008409	-0.016950
0.43750000	0.0410714	-0.0239955	-0.033378	0.020663	-0.000732
0.36718750	0.0566406	0.0056362	0.014962	-0.005309	0.016218
0.48046875	0.1614397	0.0278460	0.038295	0.096687	-0.109236
0.53906250	0.1313058	0.1659040	-0.128697	0.012025	0.054618
0.71093750	0.0371094	-0.0090960	-0.059326	-0.061617	-0.031076
0.53125000	-0.0079799	-0.0807478	0.012347	-0.010418	0.015486
0.44921875	-0.0116630	-0.0317522	-0.008789	0.007950	0.000000
0.38671875	-0.0111607	-0.0160714	-0.011300	0.000402	0.000000
0.45703125	0.0071429	-0.0061384	0.010568	0.007691	-0.002302
0.46093750	0.0342634	0.0351562	0.001988	0.005568	0.035261
0.73828125	0.0527344	0.0965960	-0.012660	-0.018109	-0.150774
0.59765625	0.0304687	0.0774553	-0.012556	-0.041212	0.146589
0.71375000	0.0060268	-0.0338170	-0.019880	-0.024452	-0.011405
0.51171875	-0.0079799	-0.0932478	-0.001569	-0.002612	0.015695
0.43750000	-0.0044085	-0.0244978	0.009312	-0.004334	-0.033064
0.37500000	-0.0677456	-0.1410714	0.003557	-0.121397	-0.143032
0.45703125	-0.0024554	0.0328125	0.024484	0.014522	-0.005650
0.51171875	-0.0028460	0.0307478	-0.024170	-0.036993	0.026681
0.75390625	-0.1251674	0.0778460	-0.168248	-0.019142	-0.088518
0.68359375	0.0162947	0.0053571	-0.122001	-0.059694	0.078265
0.70703125	-0.1199218	-0.0175781	-0.155797	0.070226	0.024379
0.44921875	-0.0365513	-0.1284040	-0.030866	-0.027522	0.101597
0.41796875	-0.0125000	0.0001116	0.000628	0.019687	-0.008161
0.15234375	-0.0695313	-0.1944197	0.006906	0.062679	0.108608
0.29296875	-0.1353237	-0.0140067	0.060582	0.000373	0.015695
0.27734375	-0.1326451	-0.0152344	0.019357	0.018683	0.025635
0.32031250	-0.0872768	0.0389509	0.044155	-0.000804	-0.024902
0.34765625	-0.1336496	-0.0653460	0.070103	-0.026145	-0.042585
0.26171875	-0.1049665	0.0693639	0.164062	0.038772	-0.000419
0.26562500	0.0065848	0.0437500	0.130371	0.086556	0.090193
0.43750000	-0.0600446	-0.1228795	-0.031076	-0.157902	-0.156948
0.07421875	-0.0148995	-0.0127790	0.015904	0.030048	0.013079

(The coefficients are given after rescaling the x and y variables to the square [-1, 1] [-1, 1], and the gray level z to [0, 1]).

<u>Step 2</u> The required accuracy ε is chosen to be 5 gray levels, k is fixed to be 2, and the grid on each segment is chosen to contain the only point - the center of this segment. Thus the Taylor polynomials computed on this step are identical to the approximating polynomials found on the step 1.

The 6 digits accuracy with which the coefficients of these polynomials are given in the array P above is excessive, and the coefficients are rounded off up to 8 bits in degree 0, up to 7 bits in degree 1 and up to 6 bits in degree 2. The corresponding binary array is the intermediate compressed image. It is approximately represented by the following digital array P' (corresponding to the same piece S of the picture, as the above array P).

0.37500000	0.0000000	0.0000000	0.000000	0.000000	0.000000
0.35937500	-0.0078125	-0.0234375	0.000000	0.000000	0.000000
0.31640625	-0.0078125	0.0000000	0.000000	0.000000	0.031250
0.39843750	-0.0078125	0.0937500	0.000000	0.000000	0.046875
0.74218750	-0.0234375	0.1640625	0.000000	0.000000	-0.046875
0.70703125	0.0234375	-0.2187500	0.000000	-0.015625	-0.031250
0.50781250	-0.0078125	-0.0312500	0.000000	0.000000	0.000000
0.43359375	0.0000000	-0.0156250	0.000000	0.000000	0.000000
0.33593750	-0.0312500	0.0000000	0.000000	0.015625	0.015625
0.37109375	0.0156250	-0.0156250	0.000000	0.000000	-0.015625
0.32421875	0.0078125	0.0000000	0.015625	0.000000	0.000000
0.38671875	0.0000000	0.0781250	0.000000	-0.015625	0.031250
0.63671875	-0.0156250	0.1953125	0.000000	0.000000	0.062500
0.81250000	0.0156250	-0.2265625	0.000000	0.015625	-0.203125
0.49218750	-0.0234375	-0.0156250	-0.015625	0.015625	0.000000
0.43750000	0.0000000	-0.0390625	0.000000	0.000000	0.000000
0.32031250	0.0078125	0.0781250	0.000000	0.000000	0.000000
0.40234375	-0.0234375	-0.0078125	-0.046875	-0.015625	0.000000
0.40234375	0.0000000	-0.0312500	-0.046875	0.015625	-0.015625
0.33984375	-0.0312500	0.0546875	0.000000	0.000000	0.046875
0.63281250	-0.0390625	0.1718750	-0.031250	0.000000	0.000000
0.80468750	-0.0703125	-0.2031250	-0.031250	0.015625	-0.218750
0.45703125	0.0000000	0.0000000	0.000000	0.000000	0.000000
0.41015625	0.0000000	-0.0312500	0.015625	0.015625	0.000000
0.33203125	-0.0078125	0.0156250	0.015625	-0.046875	0.000000
0.21093750	0.1171875	-0.0937500	0.046875	0.140625	0.062500
0.25781250	0.0859375	0.0000000	0.078125	-0.078125	-0.046875
0.26562500	-0.0156250	0.0312500	0.062500	0.000000	0.015625
0.47656250	-0.0156250	0.2109375	0.062500	0.000000	0.015625
0.57812500	-0.0234375	-0.1171875	0.000000	0.062500	0.015625
0.44531250	0.0000000	-0.0156250	0.000000	-0.015625	0.000000
0.41015625	0.0000000	-0.0312500	-0.015625	0.000000	-0.015625
0.43750000	0.0390625	-0.0234375	-0.031250	0.015625	0.000000
0.36718750	0.0546875	0.0000000	0.000000	0.000000	0.015625
0.48437500	0.1562500	0.0234375	0.031250	0.093750	-0.093750
0.53906250	0.1250000	0.1640625	-0.125000	0.000000	0.046875
0.71093750	0.0312500	-0.0078125	-0.046875	-0.046875	-0.015625
0.53125000	-0.0078125	-0.0781250	0.000000	0.000000	0.000000
0.44921875	-0.0078125	-0.0312500	0.000000	0.000000	0.000000
0.38671875	-0.0078125	-0.0156250	0.000000	0.000000	0.000000
0.45703125	0.0000000	0.0000000	0.000000	0.000000	0.000000
0.46093750	0.0312500	0.0312500	0.000000	0.000000	0.031250
0.73828125	0.0468750	0.0937500	0.000000	-0.015625	-0.140625
0.59765625	0.0234375	0.0703125	0.000000	-0.031250	0.140625
0.71875000	0.0000000	-0.0312500	-0.015625	-0.015625	0.000000
0.51171875	-0.0078125	-0.0859375	0.000000	0.000000	0.015625
0.43750000	0.0000000	-0.0234375	0.000000	0.000000	-0.031250
0.37500000	-0.0625000	-0.1406250	0.000000	-0.109375	-0.140625
0.45703125	0.0000000	0.0312500	0.015625	0.000000	0.000000
0.51171875	0.0000000	0.0234375	-0.015625	-0.031250	0.015625
0.75390625	-0.1250000	0.0703125	-0.156250	-0.015625	-0.078125
0.68359375	0.0156250	0.0000000	-0.109375	-0.046875	0.078125
0.70703125	-0.1171875	-0.0156250	-0.140625	0.062500	0.015625
0.44921875	-0.0312500	-0.1250000	-0.015625	-0.015625	0.093750
0.41796875	-0.0078125	0.0000000	0.000000	0.015625	0.000000
0.15234375	-0.0625000	-0.1875000	0.000000	0.062500	0.093750
0.29296875	-0.1328125	-0.0078125	0.046875	0.000000	0.015625
0.27734375	-0.1250000	-0.0078125	0.015625	0.015625	0.015625
0.32031250	-0.0859375	0.0312500	0.031250	0.000000	-0.015625
0.34765625	-0.1328125	-0.0625000	0.062500	-0.015625	-0.031250
0.26171875	-0.1015625	0.0625000	0.156250	0.031250	0.000000
0.26562500	0.0000000	0.0390625	0.125000	0.078125	0.078125
0.43750000	-0.0546875	-0.1171875	-0.015625	-0.156250	-0.156250
0.07421875	-0.0078125	-0.0078125	0.015625	0.015625	0.000000

The compression ratio is $512*512*8$ bits/$86*86*(8 + 2*7 + 3*6)$ bits ≈ 6.7.

The final image is obtained by computing the values of the Taylor polynomials, representing the intermediate image, at each pixel of the corresponding segment. The part S' of the obtained array, representing the final image (and corresponding to the piece S of the initial picture), is the following array 9.

137	137	137	136	138	129	138	134	140	136	135	134	130	139	135	129	134
131	138	139	127	132	129	134	131	133	130	125	130	129	127	131	127	134
128	135	133	144	135	142	143	142	143	150	148	154	153	149			

149	149	143	154	145	142	135	133	118	120	127	110	98	82	86	74	63
65	63	61	55	67	72	66	73	69	73	74	73	73	70	78	80	76
75	73	72	80	80	78	77	75	75	76	78	78	75	78			

78	79	78	78	73	75	73	75	79	79	89	77	81	86	90	86	78
89	89	92	95	92	90	92	91	101	97	92	89	94	104	100	101	96
104	98	96	98	100	95	106	108	102	103	100	100	96	99			

104	91	98	95	105	101	98	101	97	102	97	97	105	104	105	103	95
102	97	105	102	102	103	103	99	103	96	106	102	99	102	101	106	100
104	103	101	102	100	100	100	104	108	103	104	99	104	99			

105	106	101	105	105	105	101	109	106	104	102	105	104	105	102	104	112
102	99	110	104	101	100	102	102	98	100	103	103	99	98	100	101	97
107	105	104	106	101	100	100	101	99	102	97	98	99	103			

105	103	101	103	101	105	104	109	102	99	102	103	96	96	103	108	109
114	104	102	102	97	99	97	100	102	98	101	102	102	99	98	99	97
100	102	93	100	105	105	99	97	96	102	96	101	104	94			

95	103	99	95	99	95	102	93	97	99	92	96	95	102	95	101	90
94	93	89	86	93	91	90	89	86	83	85	87	74	72	73	71	82
83	88	90	102	106	111	110	116	118	125	128	126	127	136			

136	136	141	135	142	141	131	130	121	122	129	122	130	125	123	131	125
131	129	126	131	130	128	128	135	133	129	132	129	127	124	133	127	126
128	127	129	126	127	127	126	124	124	136	129	129	128	127			

129	134	129	136	135	134	129	130	131	136	136	136	132	132	132	127	131
134	134	161	181	189	198	201	206	207	206	210	212	213	211	213	215	210
204	197	183	156	131	96	77	77	71	74	76	83	73	85			

87	89	91	88	90	90	94	93	101	93	88	88	92	96	92	87	94
93	98	101	92	90	94	90	89	92	94	88	93	88	91	90	95	93
100	86	93	97	94	97	96	97	93	95	94	94	87	86			

100	92	97	96	95	101	95	97	93	89	101	95	97	97	88	94	82
91	84	88	89	84	91	93	107	139	144	147	149	148	130	99	137	137
137	136	138	129	138	134	140	136	135	134	130	139	135	129			

134	131	138	139	127	132	129	134	131	133	130	125	130	129	127	131	127
134	128	135	133	144	135	142	143	142	143	150	148	154	153	149	149	149
143	154	145	142	135	133	118	120	127	110	98	82	86	74			

63 65 63 61 55 67 72 66 73 69 73 74 73 73 70 78 80
76 75 73 72 80 80 78 77 75 75 76 78 78 75 78 78 79
78 78 73 75 73 75 79 79 89 77 81 86 90 86

78 89 89 92 95 92 90 92 91 101 97 92 89 94 104 100 101
96 104 98 96 98 100 95 106 108 102 103 100 100 96 99 104 91
98 95 105 101 98 101 97 102 97 97 105 104 105 103

95 102 97 105 102 102 103 103 99 103 96 106 102 99 102 101 106
100 104 103 101 102 100 100 100 104 108 103 104 99 104 99 105 106
101 105 105 105 101 109 106 104 102 105 104 105 102 104

112 102 99 110 104 101 100 102 102 98 100 103 103 99 98 100 101
97 107 105 104 106 101 100 100 101 99 102 97 98 99 103 105 103
101 103 101 105 104 109 102 99 102 103 96 96 103 108

109 114 104 102 102 97 99 97 100 102 98 101 102 102 99 98 99
97 100 102 93 100 105 105 99 97 96 102 96 101 104 94 95 103
99 95 99 95 102 93 97 99 92 96 95 102 95 101

90 94 93 89 86 93 91 90 89 86 83 85 87 74 72 73 71
82 83 88 90 102 106 111 110 116 118 125 128 126 127 136 136 136
141 135 142 141 131 130 121 122 129 122 130 125 123 131

125 131 129 126 131 130 128 128 135 133 129 132 129 127 124 133 127
126 128 127 129 126 127 127 126 124 124 136 129 129 128 127 129 134
129 136 135 134 129 130 131 136 136 136 132 132 132 127

131 134 134 161 181 189 198 201 206 207 206 210 212 213 211 213 215
210 204 197 183 156 131 96 77 77 71 74 76 83 73 85 87 89
91 88 90 90 94 93 101 93 88 88 92 96 92 87

94 93 98 101 92 90 94 90 89 92 94 88 93 88 91 90 95
93 100 86 93 97 94 97 96 97 93 95 94 94 87 86 100 92
97 96 95 101 95 97 93 89 101 95 97 97 88 94

82 91 84 88 89 84 91 93 107 139 144 147 149 148 130 99 137
137 137 136 138 129 138 134 140 136 135 134 130 139 135 129 134 131
138 139 127 132 129 134 131 133 130 125 130 129 127 131

127 134 128 135 133 144 135 142 143 142 143 150 148 154 153 149 149
149 143 154 145 142 135 133 118 120 127 110 98 82 86 74 63 65
63 61 55 67 72 66 73 69 73 74 73 73 70 78

80 76 75 73 72 80 80 78 77 75 75 76 78 78 75 78 78
79 78 78 73 75 73 75 79 79 89 77 81 86 90 86 78 89
89 92 95 92 90 92 91 101 97 92 89 94 104 100

101 96 104 98 96 98 100 95 106 108 102 103 100 100 96 99 104
91 98 95 105 101 98 101 97 102 97 97 105 104 105 103 95 102
97 105 102 102 103 103 99 103 96 106 102 99 102 101

106 100 104 103 101 102 100 100 100 104 108 103 104 99 104 99 105
106 101 105 105 105 101 109 106 104 102 105 104 105 102 104 112 102
99 110 104 101 100 102 102 98 100 103 103 99 98 100

101 97 107 105 104 106 101 100 100 101 99 102 97 98 99 103 105
103 101 103 101 105 104 109 102 99 102 103 96 96 103 108 109 114
104 102 102 97 99 97 100 102 98 101 102 102 99 98

99 97 100 102 93 100 105 105 99 97 96 102 96 101 104 94 95
103 99 95 99 95 102 93 97 99 92 96 95 102 95 101 90 94
93 89 86 93 91 90 89 86 83 85 87 74 72 73

71 82 83 88 90 102 106 111 110 116 118 125 128 126 127 136 136
136 141 135 142 141 131 130 121 122 129 122 130 125 123 131 125 131
129 126 131 130 128 128 135 133 129 132 129 127 124 133

127 126 128 127 129 126 127 127 126 124 124 136 129 129 128 127 129
134 129 136 135 134 129 130 131 136 136 136 132 132 132 127 131 134
134 161 181 189 198 201 206 207 206 210 212 213 211 213

215 210 204 197 183 156 131 96 77 77 71 74 76 83 73 85 87
89 91 88 90 90 94 93 101 93 88 88 92 96 92 87 94 93
98 101 92 90 94 90 89 92 94 88 93 88 91 90

95 93 100 86 93 97 94 97 96 97 93 95 94 94 87 86 100
92 97 96 95 101 95 97 93 89 101 95 97 97 88 94 82 91
84 88 89 84 91 93 107 139 144 147 149 148 130 99

137 137 137 136 138 129 138 134 140 136 135 134 130 139 135 129 134
131 138 139 127 132 129 134 131 133 130 125 130 129 127 131 127 134
128 135 133 144 135 142 143 142 143 150 148 154 153 149

149 149 143 154 145 142 135 133 118 120 127 110 98 82 86 74 63
65 63 61 55 67 72 66 73 69 73 74 73 73 70 78 80 76
75 73 72 80 80 78 77 75 75 76 78 78 75 78

78 79 78 78 73 75 73 75 79 79 89 77 81 86 90 86 78
89 89 92 95 92 90 92 91 101 97 92 89 94 104 100 101 96
104 98 96 98 100 95 106 108 102 103 100 100 96 99

104 91 98 95 105 101 98 101 97 102 97 97 105 104 105 103 95
102 97 105 102 102 103 103 99 103 96 106 102 99 102 101 106 100
104 103 101 102 100 100 100 104 108 103 104 99 104 99

105 106 101 105 105 105 101 109 106 104 102 105 104 105 102 104 112
102 99 110 104 101 100 102 102 98 100 103 103 99 98 100 101 97
107 105 104 106 101 100 100 101 99 102 97 98 99 103

105 103 101 103 101 105 104 109 102 99 102 103 96 96 103 108 109
114 104 102 102 97 99 97 100 102 98 101 102 102 99 98 99 97
100 102 93 100 105 105 99 97 96 102 96 101 104 94

95 103 99 95 99 95 102 93 97 99 92 96 95 102 95 101 90
94 93 89 86 93 91 90 89 86 83 85 87 74 72 73 71 82
83 88 90 102 106 111 110 116 118 125 128 126 127 136

136 136 141 135 142 141 131 130 121 122 129 122 130 125 123 131 125
131 129 126 131 130 128 128 135 133 129 132 129 127 124 133 127 126
128 127 129 126 127 127 126 124 124 136 129 129 128 127

129 134 129 136 135 134 129 130 131 136 136 136 132 132 132 127 131
134 134 161 181 189 198 201 206 207 206 210 212 213 211 213 215 210
204 197 183 156 131 96 77 77 71 74 76 83 73 85

87 89 91 88 90 90 94 93 101 93 88 88 92 96 92 87 94
93 98 101 92 90 94 90 89 92 94 88 93 88 91 90 95 93
100 86 93 97 94 97 96 97 93 95 94 94 87 86

100 92 97 96 95 101 95 97 93 89 101 95 97 97 88 94 82
91 84 88 89 84 91 93 107 139 144 147 149 148 130 99 137 137
137 136 138 129 138 134 140 136 135 134 130 139 135 129

134 131 138 139 127 132 129 134 131 133 130 125 130 129 127 131 127
134 128 135 133 144 135 142 143 142 143 150 148 154 153 149 149 149
143 154 145 142 135 133 118 120 127 110 98 82 86 74

63 65 63 61 55 67 72 66 73 69 73 74 73 73 70 78 80
76 75 73 72 80 80 78 77 75 75 76 78 78 75 78 78 79
78 78 73 75 73 75 79 79 89 77 81 86 90 86

78 89 89 92 95 92 90 92 91 101 97 92 89 94 104 100 101
96 104 98 96 98 100 95 106 108 102 103 100 100 96 99 104 91
98 95 105 101 98 101 97 102 97 97 105 104 105 103

95 102 97 105 102 102 103 103 99 103 96 106 102 99 102 101 106
100 104 103 101 102 100 100 100 104 108 103 104 99 104 99 105 106
101 105 105 105 101 109 106 104 102 105 104 105 102 104

112 102 99 110 104 101 100 102 102 98 100 103 103 99 98 100 101
97 107 105 104 106 101 100 100 101 99 102 97 98 99 103 105 103
101 103 101 105 104 109 102 99 102 103 96 96 103 108

The picture representing the final image is shown in Fig. 4b.

Example 3 (Rotation of a picture)

The object picture is the same as in the Example 2. The required operation is the rotation by 90° in the counterclockwise direction (Fig. 5a represents the result of a rotation of the object picture).

The array of the gray levels of the rotated piece S' of the object picture is the following array 10.

60	66	75	86	100	117	114	112	111	111	112	114	118	118	118	118	118
118	118	119	119	117	113	107	92	88	84	81	80	79	68	67	67	66
65	65	80	84	88	92	95	99	97	97	97	97	97	97			
58	64	73	84	98	115	117	115	114	114	115	117	118	118	118	118	118
118	117	118	117	114	110	104	90	87	85	83	83	83	75	74	73	73
72	71	80	83	86	90	93	96	97	97	97	97	97	97			
56	62	71	82	96	113	119	118	117	117	118	119	118	118	118	118	118
118	116	117	115	112	107	101	88	86	85	85	86	88	81	81	80	79
79	78	80	83	86	89	91	94	97	97	97	97	97	97			
56	62	70	82	96	112	122	120	119	119	120	122	118	118	118	118	118
118	115	115	114	110	105	98	86	86	86	87	90	93	88	87	87	86
85	85	81	83	86	88	91	93	97	97	97	97	97	97			
56	62	71	82	96	113	125	123	122	122	123	125	118	118	118	118	118
118	114	114	112	108	102	94	84	85	87	89	93	97	95	94	93	93
92	91	83	85	87	89	91	93	97	97	97	97	97	97			
57	63	72	83	97	114	127	126	125	125	126	127	118	118	118	118	118
118	114	113	110	106	99	91	82	84	87	91	96	102	101	101	100	99
99	98	86	87	89	91	92	94	97	97	97	97	97	97			
50	60	71	82	95	109	132	131	130	128	124	120	125	122	119	117	114
111	109	105	100	95	91	86	94	89	86	86	89	94	95	101	105	106
104	100	100	99	97	96	95	93	96	97	98	98	99	100			
48	58	68	80	92	105	130	130	130	128	126	123	124	121	118	116	113
110	108	103	98	94	89	84	89	80	73	69	68	69	93	100	104	105
104	100	100	99	98	96	95	94	94	95	96	96	97	98			
48	57	67	78	90	102	128	130	130	130	128	126	125	122	119	117	114
111	107	102	97	93	88	83	88	74	64	56	50	48	92	99	103	105
104	100	100	99	97	96	95	93	92	93	94	94	95	96			
48	57	67	77	88	101	128	130	132	132	132	130	127	125	122	119	117
114	107	102	97	93	88	83	90	72	58	46	36	30	90	97	102	104
104	101	99	97	96	95	93	92	90	91	92	92	93	94			
50	58	67	77	88	100	129	132	134	136	136	136	132	129	126	124	121
118	108	103	98	94	89	84	95	74	55	39	26	15	88	96	101	104
104	101	96	95	94	92	91	90	88	89	90	90	91	92			
52	60	68	78	88	100	130	134	138	140	141	142	138	135	133	130	127
125	109	105	100	95	91	86	104	79	56	36	19	4	86	95	100	104
104	102	93	92	91	89	88	87	86	87	88	88	89	90			

61 65 70 77 87 97 112 140 158 168 168 160 157 152 147 141 136
131 125 114 106 99 94 91 105 83 66 53 44 40 97 103 107 108
107 102 88 86 84 84 84 85 86 87 87 88 89 89

65 69 75 82 91 102 126 154 173 183 184 176 180 175 170 166 161
156 144 131 120 111 103 97 105 85 70 59 53 51 97 103 106 107
105 100 88 86 84 84 84 85 82 83 84 84 85 86

69 73 78 85 95 105 136 164 183 194 195 188 195 190 186 182 178
173 158 142 129 117 106 98 102 84 71 63 59 59 96 102 105 105
103 97 88 86 84 84 84 85 81 81 82 83 83 84

71 75 81 88 97 108 140 169 189 200 202 195 201 198 194 190 186
183 167 148 132 117 104 93 96 81 70 64 62 65 95 100 102 102
99 94 88 86 84 84 84 85 81 81 82 83 83 84

73 77 83 90 99 110 141 170 190 202 204 198 200 197 194 190 187
184 170 149 130 113 97 83 88 75 67 63 63 68 92 97 99 98
95 89 88 86 84 84 84 85 82 83 84 84 85 86

74 78 83 91 100 111 137 166 187 199 202 196 191 188 185 183 180
177 168 144 123 103 84 68 77 66 60 58 61 68 89 93 95 94
90 83 88 86 84 84 84 85 86 87 87 88 89 89

83 86 93 103 117 134 181 189 190 186 175 158 174 169 165 161 157
152 116 119 116 105 87 62 73 67 65 66 71 80 78 81 83 86
89 91 92 91 89 88 87 86 90 90 91 92 92 93

80 84 91 102 116 134 169 178 181 178 168 152 161 158 155 151 148
145 124 128 124 114 96 71 74 68 66 67 72 80 77 80 83 85
88 91 94 93 92 92 91 90 92 93 94 94 95 96

76 80 87 98 113 131 161 172 176 174 166 151 157 155 152 150 147
145 136 139 136 125 107 82 76 70 68 69 74 82 79 82 85 87
90 93 97 97 97 97 97 96 98 98 99 100 100 101

69 74 82 93 108 127 158 170 175 174 168 154 161 159 158 156 155
153 150 153 150 139 121 96 78 72 70 72 76 85 84 87 89 92
95 97 103 103 103 104 104 104 106 106 107 108 108 109

61 66 74 86 102 121 159 172 179 180 174 162 173 172 171 171 170
169 166 170 166 156 138 113 82 76 74 75 80 88 91 94 97 99
102 105 110 111 112 112 113 114 116 117 118 118 119 120

51 56 65 78 94 113 164 179 187 189 185 174 192 193 193 193 193
194 186 189 186 175 157 132 86 80 78 80 84 93 101 104 107 109
112 115 119 121 122 123 124 125 130 130 131 132 132 133

55 48 50 61 81 110 126 156 179 193 200 198 191 191 191 190 188
185 188 187 184 178 169 158 88 82 80 81 86 94 111 118 123 127
128 128 129 130 132 133 134 136 142 144 146 148 150 152

63 55 56 66 85 113 127 156 177 189 194 191 187 188 188 188 186
184 185 186 184 179 172 162 104 98 96 97 102 110 126 133 138 141
143 143 139 140 141 143 144 145 161 163 165 167 169 171

70 62 62 71 89 116 130 156 175 186 189 184 183 185 185 185 184
182 182 184 184 180 174 166 121 115 113 114 119 128 141 148 153 156
158 157 152 153 154 156 157 158 178 180 182 184 186 188

78 68 68 76 93 119 133 158 175 184 185 178 179 181 182 183 182
180 178 182 182 180 176 168 139 133 131 132 137 146 155 162 167 171
172 172 168 170 171 172 174 175 192 194 196 198 200 202

85 75 73 81 97 122 137 160 175 183 182 173 176 178 180 180 180
179 173 178 180 179 176 170 158 152 150 151 156 164 170 177 182 185
187 187 189 190 191 193 194 195 203 205 207 209 211 213

93 82 79 86 101 125 142 163 177 182 180 169 172 175 177 178 178
177 168 174 177 178 176 171 178 172 170 171 176 184 185 192 197 200
202 201 212 214 215 216 218 219 212 214 216 218 220 222

83 74 73 78 91 111 153 156 158 160 161 160 150 151 152 152 153
154 151 152 152 153 154 154 160 166 173 179 185 192 187 198 207 214
219 223 221 221 220 220 220 220 230 227 224 221 217 214

83 72 68 72 82 100 130 134 137 139 140 140 141 142 143 143 144
145 144 145 146 146 147 148 152 157 162 166 171 176 196 206 214 221
226 229 226 225 224 224 223 222 214 211 208 206 203 200

87 74 68 69 78 93 113 117 121 123 125 125 133 134 134 135 136
136 138 138 139 140 140 141 146 149 152 155 157 160 192 202 210 216
220 223 219 218 217 216 215 213 196 193 191 189 187 185

96 81 73 72 78 91 101 106 110 113 115 116 126 126 127 128 128
129 131 132 132 133 134 134 140 141 143 144 145 146 176 185 193 198
202 205 201 199 198 196 195 193 176 174 172 171 169 167

110 92 82 78 82 93 95 100 104 108 110 112 119 120 121 121 122
123 124 125 126 126 127 128 136 135 134 134 133 132 147 156 163 168
172 174 171 169 167 165 163 161 154 153 152 150 149 148

127 108 95 89 91 100 94 99 104 108 111 113 114 114 115 116 116
117 118 118 119 120 120 121 132 129 127 125 122 120 106 114 121 126
129 130 130 127 125 122 120 117 131 130 129 128 127 127

124 119 113 107 100 91 104 105 107 109 111 112 112 112 112 112 112
112 121 122 122 123 124 124 121 120 119 118 117 116 118 118 118 118
118 118 119 124 128 131 133 134 135 136 136 137 138 138

120 120 119 117 114 110 105 106 107 109 110 111 114 114 114 114 114
114 118 119 120 120 121 122 119 118 117 117 116 115 118 118 118 118
118 118 119 123 127 129 131 132 132 133 134 134 135 136

108 112 116 118 120 120 106 107 108 108 109 110 114 114 114 114 114
114 116 116 117 118 118 119 116 116 116 116 115 115 118 118 118 118
118 118 118 122 125 128 129 129 130 130 131 132 132 133

87 96 103 110 116 121 107 107 108 108 109 109 112 112 112 112 112
112 113 114 114 115 116 116 114 114 114 114 115 115 118 118 118 118
118 118 118 122 124 126 127 127 127 128 128 129 130 130

57 70 82 94 104 114 108 108 108 108 108 108 108 108 108 108 108
108 110 111 112 112 113 114 111 112 113 113 114 115 118 118 118 118
118 118 118 121 123 125 125 125 124 125 126 126 127 128

18 36 52 68 83 97 109 109 108 108 107 107 102 102 102 102 102
102 108 108 109 110 110 111 109 110 111 112 113 114 118 118 118 118
118 118 118 120 122 123 123 122 122 122 123 124 124 125

20 20 21 23 25 29 72 82 92 102 111 121 108 106 103 101 98
96 102 102 103 104 104 105 107 109 110 110 109 107 113 112 112 113
115 118 121 121 121 121 121 121 115 115 115 115 115 115

20 20 20 22 24 27 50 58 66 74 82 90 104 105 106 106 107
108 100 101 102 102 103 104 106 108 109 109 108 106 111 110 110 110
112 114 118 118 118 118 118 118 114 114 114 114 114 114

21 20 20 21 23 25 33 39 46 52 58 64 93 96 100 104 108
111 99 100 100 101 102 102 104 106 107 107 106 104 110 108 107 108
109 111 115 115 115 115 115 115 113 113 113 113 113 113

21 20 20 20 21 24 22 26 30 35 39 44 73 80 87 93 100
107 98 98 99 100 100 101 102 104 104 104 104 102 108 106 105 105
105 107 111 111 111 111 111 111 111 111 111 111 111 111

22 20 19 19 20 22 15 18 21 23 26 29 45 55 65 75 85
95 96 97 98 98 99 100 98 100 101 101 100 98 106 104 102 102
102 103 108 108 108 108 108 108 110 110 110 110 110 110

22 20 19 18 19 20 14 15 16 17 18 19 9 22 35 49 62
75 95 96 96 97 98 98 94 96 96 96 96 94 105 102 100 99
99 99 105 105 105 105 105 105 109 109 109 109 109 109

The above rotation acts on the Taylor polynomials, representing the intermediate image, obtained in the Example 2, as follows: let the 6 x 6 pixel square segments, into which the original picture has been subdivided, be indexed by two indices i and j, in such a way that the middle segment has indices 0, 0. Denote the Taylor polynomial corresponding to the segment i, j by p_{ij}. Then:

a. The indices i, j of each p_{ij} are replaced by -j, i

b. x is replaced by y, and y by -x.

Using the notations already used in discussing processing, $F(p_{ij}(x,y))=$

$p_{-j,\,i}(y, -x)$.

The result of the application of the corresponding subroutine to the Taylor polynomials in the intermediate range, obtained in the Example 2, is the intermediate range of the rotated picture. Its part P' corresponding to the rotated piece S', is the following array 11.

0.29296875	-0.0078125	0.0078125	0.015625	-0.000000	0.015625
0.45703125	0.0312500	-0.0312500	0.000000	-0.000000	0.000000
0.45703125	0.0000000	-0.0000000	0.000000	-0.000000	0.000000
0.43750000	-0.0234375	0.0234375	0.000000	-0.015625	0.000000
0.33203125	0.0156250	-0.0156250	0.000000	0.046875	0.000000
0.32031250	0.0781250	-0.0781250	0.000000	-0.000000	0.000000
0.33593750	0.0000000	-0.0000000	0.015625	-0.015625	0.015625
0.37500000	0.0000000	-0.0000000	0.000000	-0.000000	0.000000
0.27734375	-0.0078125	0.0078125	0.015625	-0.015625	0.015625
0.51171875	0.0234375	-0.0234375	0.015625	0.031250	0.015625
0.46093750	0.0312500	-0.0312500	0.031250	-0.000000	0.031250
0.36718750	0.0000000	-0.0000000	0.015625	-0.000000	0.015625
0.21093750	-0.0937500	0.0937500	0.062500	-0.140625	0.062500
0.40234375	-0.0078125	0.0078125	0.000000	0.015625	0.000000
0.37109375	-0.0156250	0.0156250	-0.015625	-0.000000	-0.015625
0.35937500	-0.0234375	0.0234375	0.000000	-0.000000	0.000000
0.32031250	0.0312500	-0.0312500	-0.015625	-0.000000	-0.015625
0.75390625	0.0703125	-0.0703125	-0.078125	0.015625	-0.078125
0.73828125	0.0937500	-0.0937500	-0.140625	0.015625	-0.140625
0.48437500	0.0234375	-0.0234375	-0.093750	-0.093750	-0.093750
0.25781250	0.0000000	-0.0000000	-0.046875	0.078125	-0.046875
0.40234375	-0.0312500	0.0312500	-0.015625	-0.015625	-0.015625
0.32421875	0.0000000	-0.0000000	0.000000	-0.000000	0.000000
0.31640625	0.0000000	-0.0000000	0.031250	-0.000000	0.031250
0.34765625	-0.0625000	0.0625000	-0.031250	0.015625	-0.031250
0.68359375	0.0000000	-0.0000000	0.078125	0.046875	0.078125
0.59765625	0.0703125	-0.0703125	0.140625	0.031250	0.140625
0.53906250	0.1640625	-0.1640625	0.046875	-0.000000	0.046875
0.26562500	0.0312500	-0.0312500	0.015625	-0.000000	0.015625
0.33984375	0.0546875	-0.0546875	0.046875	-0.000000	0.046875
0.38671875	0.0781250	-0.0781250	0.031250	0.015625	0.031250
0.39843750	0.0937500	-0.0937500	0.046875	-0.000000	0.046875
0.26171875	0.0625000	-0.0625000	0.000000	-0.031250	0.000000
0.70703125	-0.0156250	0.0156250	0.015625	-0.062500	0.015625
0.71875000	-0.0312500	0.0312500	0.000000	0.015625	0.000000
0.71093750	-0.0078125	0.0078125	-0.015625	0.046875	-0.015625
0.47656250	0.2109375	-0.2109375	0.015625	-0.000000	0.015625
0.63281250	0.1718750	-0.1718750	0.000000	-0.000000	0.000000
0.63671875	0.1953125	-0.1953125	0.062500	-0.000000	0.062500
0.74218750	0.1640625	-0.1640625	-0.046875	-0.000000	-0.046875
0.26562500	0.0390625	-0.0390625	0.078125	-0.078125	0.078125
0.44921875	-0.1250000	0.1250000	0.093750	0.015625	0.093750
0.51171875	-0.0859375	0.0859375	0.015625	-0.000000	0.015625
0.53125000	-0.0781250	0.0781250	0.000000	-0.000000	0.000000
0.57812500	-0.1171875	0.1171875	0.015625	-0.062500	0.015625
0.80468750	-0.2031250	0.2031250	-0.218750	-0.015625	-0.218750
0.81250000	-0.2265625	0.2265625	-0.203125	-0.015625	-0.203125
0.70703125	-0.2187500	0.2187500	-0.031250	0.015625	-0.031250
0.43750000	-0.1171875	0.1171875	-0.156250	0.156250	-0.156250
0.41796875	0.0000000	-0.0000000	0.000000	-0.015625	0.000000
0.43750000	-0.0234375	0.0234375	-0.031250	-0.000000	-0.031250
0.44921875	-0.0312500	0.0312500	0.000000	-0.000000	0.000000
0.44531250	-0.0156250	0.0156250	0.000000	0.015625	0.000000
0.45703125	0.0000000	-0.0000000	0.000000	-0.000000	0.000000
0.49218750	-0.0156250	0.0156250	0.000000	-0.015625	0.000000
0.50781250	-0.0312500	0.0312500	0.000000	-0.000000	0.000000
0.07421875	-0.0078125	0.0078125	0.000000	-0.015625	0.000000
0.15234375	-0.1875000	0.1875000	0.093750	-0.062500	0.093750
0.37500000	-0.1406250	0.1406250	-0.140625	0.109375	-0.140625
0.38671875	-0.0156250	0.0156250	0.000000	-0.000000	0.000000
0.41015625	-0.0312500	0.0312500	-0.015625	-0.000000	-0.015625
0.41015625	-0.0312500	0.0312500	0.000000	-0.015625	0.000000
0.43750000	-0.0390625	0.0390625	0.000000	-0.000000	0.000000
0.43359375	-0.0156250	0.0156250	0.000000	-0.000000	0.000000

The final image, produced from the data rotated in a compressed form, is shown in Fig. 5b.

Example 4 (Producing a negative picture)

The object picture is the same as in the Example 2. It is required to produce a negative of this picture. Under this operation each gray level value z must be replaced by $z' = 255 - z$.

The negative of the original picture is shown in Fig. 6a. The array S" of the gray levels, corresponding to the negative of the piece S, is the following.

158 158 158 158 158 158 155 157 159 161 163 165 166 169 171 171 169
166 162 159 154 146 135 122 103 84 67 53 42 33 41 55 70 88
107 128 117 119 122 125 127 130 140 141 142 144 145 146

158 158 158 158 158 158 156 158 160 162 164 166 166 170 172 172 170
166 163 160 155 147 136 123 105 86 69 55 44 35 38 52 68 86
106 128 117 120 123 125 128 131 140 141 142 144 145 146

158 158 158 158 158 158 157 159 161 163 165 167 167 171 172 172 171
167 163 161 155 147 137 123 107 88 71 57 46 37 34 49 66 84
105 127 118 121 123 126 129 131 140 141 142 144 145 146

158 158 158 158 158 158 157 159 161 163 165 167 168 171 173 173 171
168 164 161 156 148 137 124 109 90 73 59 48 39 31 47 64 83
103 126 119 121 124 127 129 132 140 141 142 144 145 146

158 158 158 158 158 158 158 160 162 164 166 168 168 172 174 174 172
168 165 162 157 149 138 125 111 92 75 61 50 41 28 44 62 81
102 125 119 122 125 127 130 133 140 141 142 144 145 146

158 158 158 158 158 158 159 161 163 165 167 169 169 173 174 174 173
169 165 163 157 149 139 125 113 94 77 63 52 43 25 41 59 79
101 124 120 123 125 128 131 133 140 141 142 144 145 146

156 159 161 162 162 161 162 161 162 163 165 168 170 170 170 170 170
170 169 165 159 151 141 130 119 110 97 80 60 36 35 33 42 62
94 138 121 123 126 128 130 133 134 137 140 144 147 150

160 162 164 164 164 163 160 160 160 162 164 167 171 171 171 171 171
171 168 164 158 151 142 131 121 111 98 81 61 37 35 32 40 60
92 135 122 124 126 128 130 132 134 137 140 144 147 150

163 165 166 167 166 164 159 159 159 160 163 166 171 171 171 171 171
171 167 163 158 151 143 132 122 112 99 83 62 39 35 31 39 59
90 133 124 126 127 129 130 132 134 137 140 144 147 150

167 169 169 169 168 166 158 157 158 159 161 164 171 171 171 171 171
171 166 163 158 152 143 133 123 114 101 84 64 40 35 31 38 57
88 130 127 128 130 131 132 133 134 137 140 144 147 150

171 172 172 172 170 168 156 156 156 158 160 163 169 169 169 169 169
169 164 162 158 152 144 134 125 115 102 85 65 41 34 30 37 56
86 128 131 132 133 133 134 135 134 137 140 144 147 150

175 175 175 174 172 169 155 155 155 156 159 162 167 167 167 167 167
167 163 161 158 152 145 136 126 116 103 87 66 43 34 29 36 54
84 125 136 136 137 137 137 137 134 137 140 144 147 150

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190 184 177 170 164 157 155 155 155 154 154 153 153 155 158 161 166
172 164 164 162 158 150 140 127 112  98  83  68  54  32  26  32  50
 81 125 137 137 137 137 137 137 137 141 144 148 152 156

190 183 176 170 163 156 151 151 151 151 151 151 148 150 152 156 160
165 166 167 165 160 153 143 127 112  97  83  68  53  36  29  35  53
 83 126 137 137 137 137 137 137 140 143 146 150 153 156

189 182 176 169 162 156 149 150 150 151 151 151 147 148 150 153 157
161 169 170 168 163 156 146 128 114  99  84  70  55  41  34  39  57
 87 129 137 137 137 137 137 137 142 145 147 150 153 156

188 182 175 168 162 155 150 151 152 153 154 155 148 149 150 153 156
160 172 172 170 166 158 148 132 117 102  88  73  58  48  41  45  62
 92 134 137 137 137 137 137 137 143 145 148 150 153 155

188 181 174 168 161 154 154 155 156 158 159 160 152 152 153 155 158
162 174 175 173 168 161 151 137 122 107  93  78  63  57  49  53  70
 99 141 137 137 137 137 137 137 143 145 147 149 151 153

187 180 174 167 160 154 160 162 163 165 167 169 158 158 159 160 163
166 177 178 176 171 164 154 144 129 114 100  85  70  68  59  63  79
108 149 137 137 137 137 137 137 142 144 145 147 149 150

176 172 167 162 158 153 161 186 207 225 240 251 215 204 196 190 187
187 175 175 173 170 167 162 161 145 127 109  91  71  63  79  95 109
123 135 139 140 140 140 140 141 148 149 151 153 157 161

175 172 169 165 162 159 166 187 205 219 229 236 211 202 196 193 192
194 184 183 181 179 175 171 169 153 136 118  99  79  70  84  98 110
122 133 138 139 140 140 141 142 146 147 149 151 155 159

174 172 170 168 166 164 169 186 199 209 216 219 202 196 192 191 192
197 189 188 186 183 180 175 174 158 141 123 104  84  76  89 100 111
121 130 137 138 139 141 142 143 145 146 148 151 154 159

171 170 170 169 168 168 169 182 191 197 200 199 189 185 184 185 188
195 190 189 187 185 181 177 175 159 142 124 105  85  82  93 103 112
121 128 136 138 139 141 142 144 145 146 148 151 154 159

167 168 169 169 170 171 166 175 181 183 181 176 172 170 171 174 180
189 188 187 185 183 179 175 173 157 140 122 103  83  89  98 106 114
120 126 135 137 139 141 143 145 146 147 149 151 155 159

163 165 167 169 171 173 161 166 167 165 160 151 150 150 153 159 167
178 182 181 179 177 173 169 167 151 134 116  97  77  95 103 109 115
119 123 134 136 139 141 144 146 148 149 151 153 157 161
```

148 151 154 157 161 164 169 171 172 172 171 169 164 158 157 162 172
187 193 184 173 159 142 123 97 93 89 87 85 84 101 107 114 121
127 134 131 133 136 139 141 144 150 151 153 154 155 157

142 145 148 150 153 156 164 166 167 167 166 164 161 152 149 151 158
171 168 159 148 134 117 98 86 83 81 79 79 79 101 108 115 121
128 135 131 134 137 139 142 145 151 152 153 155 156 157

138 141 143 145 147 149 160 161 162 162 161 160 156 144 138 138 142
152 150 141 130 116 99 80 77 76 75 75 76 77 102 109 115 122
129 135 132 135 137 140 143 145 151 153 154 155 157 158

136 138 140 141 143 145 155 157 158 158 157 155 149 135 126 123 125
132 139 131 119 105 89 69 71 71 71 73 75 78 103 109 116 123
129 136 133 135 138 141 143 146 152 153 155 156 157 159

136 137 138 140 141 142 150 152 153 153 152 150 141 124 113 107 106
111 136 127 116 102 85 66 68 69 71 73 77 81 103 110 117 123
130 137 133 136 139 141 144 147 153 154 155 157 158 159

137 138 139 140 141 141 146 147 148 148 147 146 130 111 97 88 85
87 139 131 119 105 89 69 67 70 73 77 82 87 104 111 117 124
131 137 134 137 139 142 145 147 153 155 156 157 159 160

137 137 137 137 137 137 144 145 144 141 137 130 124 99 82 72 71
78 103 110 110 102 86 61 70 71 73 75 76 78 101 110 119 126
132 138 143 141 141 143 147 153 159 147 144 148 160 180

137 137 137 137 137 137 141 142 141 138 134 128 119 94 77 69 68
75 98 107 108 100 85 62 67 69 71 73 75 77 102 111 119 127
133 139 143 141 141 143 147 153 157 148 147 155 170 193

137 137 137 137 137 137 138 139 138 136 131 125 114 89 73 65 65
72 94 104 105 99 84 62 65 67 70 72 75 77 103 112 120 127
134 139 143 141 141 143 147 153 154 149 151 162 180 206

137 137 137 137 137 137 136 137 136 133 129 122 108 85 69 61 61
70 90 100 103 97 84 62 64 67 70 73 75 78 103 112 121 128
134 140 143 141 141 143 147 153 152 149 155 168 190 220

137 137 137 137 137 137 133 134 133 130 126 120 103 80 65 57 58
67 86 97 100 96 83 62 64 67 70 74 77 80 104 113 121 129
135 141 143 141 141 143 147 153 149 150 159 175 200 233

137 137 137 137 137 137 130 131 130 128 123 117 98 75 60 54 55
64 81 94 98 94 82 63 64 68 72 76 79 83 105 114 122 129
136 141 143 141 141 143 147 153 147 151 162 182 210 246

141 138 136 133 130 128 135 132 129 125 119 113 95 79 67 60 57
59 97 103 104 101 93 81 57 64 71 77 82 86 95 115 130 139
143 142 143 144 145 146 147 148 134 165 191 211 226 236

143 140 137 135 132 129 131 129 127 123 119 114 87 71 60 53 51
53 80 87 89 87 81 70 55 61 66 70 73 75 94 115 130 140
145 144 144 145 146 146 147 148 144 173 197 216 229 237

144 141 138 136 133 130 127 127 125 123 119 115 87 72 61 55 53
56 69 77 81 81 75 66 62 66 69 71 72 73 95 116 132 142
147 147 146 146 147 147 147 147 153 181 203 220 232 238

144 141 138 136 133 130 125 125 125 123 121 117 97 82 72 66 65
68 65 74 79 80 76 68 76 78 80 80 80 78 97 118 134 145
151 151 148 148 147 147 147 147 163 189 209 225 234 239

143 140 137 135 132 129 124 125 125 125 123 121 115 101 91 86 85
89 66 77 83 85 83 76 99 99 99 97 95 92 99 121 138 149
155 156 150 149 148 148 147 146 173 197 216 229 237 240

141 138 136 133 130 128 123 125 127 127 126 125 143 129 119 115 114
118 74 86 94 97 96 91 129 128 125 122 118 113 102 125 142 154
160 161 151 150 149 148 147 146 183 205 222 233 240 241

138 140 142 143 142 141 146 150 153 154 155 155 158 153 150 147 145
144 121 121 124 128 134 142 145 142 139 136 133 130 144 155 162 164
162 155 164 145 135 134 141 158 226 228 230 231 233 235

155 157 159 159 159 158 160 163 165 167 167 167 168 164 160 158 156
155 138 139 142 147 153 161 174 170 166 162 158 154 164 173 177 177
173 164 155 141 135 139 151 172 230 231 232 234 235 236

169 171 173 173 173 172 173 175 177 178 178 177 178 173 170 167 165
164 152 153 157 162 169 177 194 189 184 179 174 169 177 183 186 183
177 166 148 138 137 145 161 187 232 233 234 235 236 237

180 182 184 185 184 183 184 187 188 188 188 187 185 180 177 174 172
172 162 164 168 173 181 190 205 199 193 187 182 176 182 187 187 182
173 160 142 136 139 152 173 203 234 235 235 235 236 236

189 191 193 193 193 192 195 197 198 198 197 195 190 186 182 180 178
177 169 171 175 181 189 199 207 200 193 187 180 173 181 183 181 174
163 147 136 135 143 159 185 219 235 235 235 235 235 235

195 197 199 199 199 198 205 207 207 207 205 203 194 190 186 184 182
181 172 175 179 186 194 204 200 192 185 177 170 162 172 172 168 159
145 128 131 135 147 168 198 237 235 235 234 234 233 233

The above operation on Taylor polynomials is the following:

$F(a_0 + a_1 x + a_2 y + a_{11} x^2 + 2a_{12} xy + a_{22} y^2) =$

$1 - a_0 - a_1 x - a_2 y - a_{11} x^2 - 2a_{12} xy - a_{22} y^2$

(in the same rescaling as above).

The corresponding subroutine, applied to the Taylor polynomials of the intermediate image obtained in the Example 2, gives the intermediate image of the negative. The part of the polynomials array P'', corresponding to the piece S'' of the negative, is the following.

0.62500000	-0.0000000	-0.0000000	-0.000000	-0.000000	-0.000000
0.64062500	0.0078125	0.0234375	-0.000000	-0.000000	-0.000000
0.68359375	0.0078125	-0.0000000	-0.000000	-0.000000	-0.031250
0.60156250	0.0078125	-0.0937500	-0.000000	-0.000000	-0.046875
0.25781250	0.0234375	-0.1640625	-0.000000	-0.000000	0.046875
0.29296875	-0.0234375	0.2187500	-0.000000	0.015625	0.031250
0.49218750	0.0078125	0.0312500	-0.000000	-0.000000	-0.000000
0.56640625	-0.0000000	0.0156250	-0.000000	-0.000000	-0.000000
0.66406250	0.0312500	-0.0000000	-0.000000	-0.015625	-0.015625
0.62890625	-0.0156250	0.0156250	-0.000000	-0.000000	0.015625
0.67578125	-0.0078125	-0.0000000	-0.015625	-0.000000	-0.000000
0.61328125	-0.0000000	-0.0781250	-0.000000	0.015625	-0.031250
0.36328125	0.0156250	-0.1953125	-0.000000	-0.000000	-0.062500
0.18750000	-0.0156250	0.2265625	-0.000000	-0.015625	0.203125
0.50781250	0.0234375	0.0156250	0.015625	-0.015625	-0.000000
0.56250000	-0.0000000	0.0390625	-0.000000	-0.000000	-0.000000
0.67968750	-0.0078125	-0.0781250	-0.000000	-0.000000	-0.000000
0.59765625	0.0234375	0.0078125	0.046875	0.015625	-0.000000
0.59765625	-0.0000000	0.0312500	0.046875	-0.015625	0.015625
0.66015625	0.0312500	-0.0546875	-0.000000	-0.000000	-0.046875
0.36718750	0.0390625	-0.1718750	0.031250	-0.000000	-0.000000
0.19531250	0.0703125	0.2031250	0.031250	-0.015625	0.218750
0.54296875	-0.0000000	-0.0000000	-0.000000	-0.000000	-0.000000
0.58984375	-0.0000000	0.0312500	-0.015625	-0.015625	-0.000000
0.66796875	0.0078125	-0.0156250	-0.015625	0.046875	-0.000000
0.78906250	-0.1171875	0.0937500	-0.046875	-0.140625	-0.062500
0.74218750	-0.0859375	-0.0000000	-0.078125	0.078125	0.046875
0.73437500	0.0156250	-0.0312500	-0.062500	-0.000000	-0.015625
0.52343750	0.0156250	-0.2109375	-0.062500	-0.000000	-0.015625
0.42187500	0.0234375	0.1171875	-0.000000	-0.062500	-0.015625
0.55468750	-0.0000000	0.0156250	-0.000000	0.015625	-0.000000
0.58984375	-0.0000000	0.0312500	0.015625	-0.000000	0.015625
0.56250000	-0.0390625	0.0234375	0.031250	-0.015625	-0.000000
0.63281250	-0.0546875	-0.0000000	-0.000000	-0.000000	-0.015625
0.51562500	-0.1562500	-0.0234375	-0.031250	-0.093750	0.093750
0.46093750	-0.1250000	-0.1640625	0.125000	-0.000000	-0.046875
0.28906250	-0.0312500	0.0078125	0.046875	0.046875	0.015625
0.46875000	0.0078125	0.0781250	-0.000000	-0.000000	-0.000000
0.55078125	0.0078125	0.0312500	-0.000000	-0.000000	-0.000000
0.61328125	0.0078125	0.0156250	-0.000000	-0.000000	-0.000000
0.54296875	-0.0000000	-0.0000000	-0.000000	-0.000000	-0.000000
0.53906250	-0.0312500	-0.0312500	-0.000000	-0.000000	-0.031250
0.26171875	-0.0468750	-0.0937500	-0.000000	0.015625	0.140625
0.40234375	-0.0234375	-0.0703125	-0.000000	0.031250	-0.140625
0.28125000	-0.0000000	0.0312500	0.015625	0.015625	-0.000000
0.48828125	0.0078125	0.0859375	-0.000000	-0.000000	-0.015625
0.56250000	-0.0000000	0.0234375	-0.000000	-0.000000	0.031250
0.62500000	0.0625000	0.1406250	-0.000000	0.109375	0.140625
0.54296875	-0.0000000	-0.0312500	-0.015625	-0.000000	-0.000000
0.48828125	-0.0000000	-0.0234375	0.015625	0.031250	-0.015625
0.24609375	0.1250000	-0.0703125	0.156250	0.015625	0.078125
0.31640625	-0.0156250	-0.0000000	0.109375	0.046875	-0.078125
0.29296875	0.1171875	0.0156250	0.140625	-0.062500	-0.015625
0.55078125	0.0312500	0.1250000	0.015625	0.015625	-0.093750
0.58203125	0.0078125	-0.0000000	-0.000000	-0.015625	-0.000000
0.84765625	0.0625000	0.1875000	-0.000000	-0.062500	-0.093750
0.70703125	0.1328125	0.0078125	-0.046875	-0.000000	-0.015625
0.72265625	0.1250000	0.0078125	-0.015625	-0.015625	-0.015625
0.67968750	0.0859375	-0.0312500	-0.031250	-0.000000	0.015625
0.65234375	0.1328125	0.0625000	-0.062500	0.015625	0.031250
0.73828125	0.1015625	-0.0625000	-0.156250	-0.031250	-0.000000
0.73437500	-0.0000000	-0.0390625	-0.125000	-0.078125	-0.078125
0.56250000	0.0546875	0.1171875	0.015625	0.156250	0.156250
0.92578125	0.0078125	0.0078125	-0.015625	-0.015625	-0.000000

The final image produced from the intermediate negative image, obtained as above, is shown in Fig. 6b.

While a number of embodiments of the invention have been discussed and illustrated, it will be understood that the invention may be carried out in a number of ways and with many modifications, adaptations, and variations, by persons skilled in the art, without departing from its spirit and from the scope of the appended claims.

CLAIMS

1 - Process for the production of images of objects, as hereinbefore defined, comprosing the steps of:

(1) Approximating the object by a model comprising at least one differentiable component.

(2) Establishing the maximum allowable error ε and the degree k of the polynomials by which the differentiable component(s) of the model are to be approximated.

(3) Constructing a grid of a suitable pitch h.

(4) Computing the coefficients of the Taylor polynomials of the aforesaid differentiable component(s) at selected points of said grid.

2 - Process according to claim 1, wherein the object is defined in a space having more than three dimensions.

3 - Process according to claim 1, wherein the object is a line.

4 - Process according to claim 1, wherein the object is a surface.

5 - Process according to claim 1, wherein the object is a solid.

6 - Process according to claim 1, wherein the model further comprises at least one non-differentiable component.

7 - Process according to claim 1, comprising carrying out the said steps at least in part concurrently.

8 - Process according to claim 1, wherein the object is defined by data which are values and/or relationships embodied in physical entities.

9 - Process according to claim 8, comprising the preliminary step of storing the data defining the object in an electronic memory.

10 - Process according to claim 1, comprising determining the parameters of the components of the model by minimizing a quantity representing an error

11 - Process according to claim 10, wherein the quantity representing an error is the quadratical error.

12 - Process according to claim 1, wherein the non-differentiable component(s) of the model embody the same discontinuities as the object, and the differentiable component(s) represent the deviations of the object lfrom the non-differentiable component.

`13 - Process according to claim 12, wherein the model has the form:

(1) $\Phi(x) = Hx_o,a,b,c,d(x) + \phi(x)$

wherein H is defined by $H(x) = a(x-x_o) + b$, if $x \geq x_o$ or $H(x) = c(x-x_o) + d$, if x is less than x_o.

13 - Process according to claim 1, wherein the model is a differentiable function of another functicn which embodies the non-differentiable characteristics of the object.

14 - Process according to claim 1, wherein each grid pitch is calculatedfrom the formula

(3) $CMh^{k+1} \leq \varepsilon$

wherein $C = 1/(k+1)!$ and M is the maximum, at each grid point, of the absolute value of the derivatives of degree k+1 of the differentiable component or components of the model.

15- Process according to claim 1, further comprising constructing an adjusted image line by applying to each differentiable component the Whitney subroutine, and minimizing the quntity W thus computed, under such constraints that the results of the minimization do not deviate from the initial data by more than the allowed error.

16 - Process according to claim 1, further comprising rounding off the coefficients of the Taylor polynomials to a maximum allowable error greater than the original one.

18 – Process according to claim 1, further comprising separating a temporary image into components of increasing fineness, constructing a grid which is sparser than the one used for obtaining said image and the pitch of which is determined by the resolution required by the lowest fineness of said components, obtaining thereforom a second temporary image, subtracting said second temporary image from the original one to obtain a first residual image, and repeating the same steps for successively finer components, correspondingly obtaining successive residual images, whereby to compute coefficients of Taylor polynomials on several grids having increasingly higher resolutions.

19 - Process according to claim 1, further comprising applying to the coefficients of the Taylor polynomials any desired known encoding method.

20 - Process according to claim 1, further comprising applying to any data obtained in carrying out the process any desired known encoding method.

21 - Process according to claim 1, further comprising constructing a final image by a procedure comprising the steps of dividing the domain, in which the temporary image has been defined, into possibly overlapping regions by means of a grid, each region being a portion of the grid around a grid node, and constructing curves representing the Taylor polynomials of degree k from the coefficients defining the temporary image at each grid node.

22 - Process according to claim 1, further comprising processing the obtained data, representing an intermediate image, by applying thereto an operator, whereby to obtain an image representing an object which is the result of applying to the original object the said operator.

COMPRESSED IMAGE PRODUCTION, STORAGE, TRANSMISSION AND PROCESSING

ABSTRACT

Images of objects are produced by:

(1) Approximating the object by a model comprising at least one differentiable component.

(2) Establishing the maximum allowable error ε and the degree k of the polynomials by which the differentiable component(s) of the model are to be approximated.

(3) Constructing a grid of a suitable pitch h.

(4) Computing the coefficients of the Taylor polynomials of the aforesaid differentiable component(s) at selected points of said grid.

2 - Process according to claim 1, wherein the object is defined in a space having more than three dimensions.

3 - Process according to claim 1, wherein the object is a line.

4 - Process according to claim 1, wherein the object is a surface.

5 - Process according to claim 1, wherein the object is a solid.

6 - Process according to claim 1, wherein the model further comprises at least one non-differentiable component.

7 - Process according to claim 1, comprising carrying out the said steps at least in part concurrently.

8 - Process according to claim 1, wherein the object is defined by data which are values and/or relationships embodied in physical entities.

9 - Process according to claim 8, comprising the preliminary step of storing the data defining the object in an electronic memory.

10 - Process for the production of images of objects, according to the claim 1, wherein said second component of the model is defined by minimizing, by a predetermined subroutine, a quantity representing the deviation from the object of a model consisting of the first and second components.

11 - Process for the production of images of objects according to claim 1, wherein the data defining the object, the data defining the model, and the data defining the images, are digital data.

12 - Process according to claim 1, wherein the model has the form:

(1) $\Phi(x) = Hx_o,a,b,c,d(x) + \phi(x)$

wherein H is defined by $H(x) = a(x-x_o) + b$, if $x \geq x_o$ or $H(x) = c(x-x_o) + d$, if x is less than x_o.

13 - Process according to claim 1, wherein the model is a differentiable function of another functicn which embodies the non-differentiable characteristics of the object.

14 - Process according to claim 1, wherein each grid pitch is calculatedfrom the formula

(3) $CMh^{k+1} \leq \varepsilon$

wherein $C = 1/(k+1)!$ and M is the maximum, at each grid point, of the absolute value of the derivatives of degree k+1 of the differentiable component or components of the model.

15- Process according to claim 1, further comprising constructing an adjusted image line by applying to each differentiable component the Whitney subroutine, and minimizing the quntity W thus computed, under such constraints that the results of the minimization do not deviate from the initial data by more than the allowed error.

16 - Process according to claim 1, further comprising rounding off the coefficients of the Taylor polynomials to a maximum allowable error greater than the original one.

17 – Process according to claim 1, further comprising separating a temporary image into components of increasing fineness, constructing a grid which is sparser than the one used for obtaining said image and the pitch of which is determined by the resolution required by the lowest fineness of said components, obtaining thereforom a second temporary image, subtracting said second temporary image from the original one to obtain a first residual image, and repeating the same steps for successively finer components, correspondingly obtaining successive residual images, whereby to compute coefficients of Taylor polynomials on several grids having increasingly higher resolutions.

18 - Process according to claim 1, further comprising applying to the coefficients of the Taylor polynomials any desired known encoding method.

19 - Process according to claim 1, further comprising applying to any data obtained in carrying out the process any desired known encoding method.

20 - Process according to claim 1, further comprising constructing a final image by a procedure comprising the steps of dividing the domain, in which the temporary image has been defined, into possibly overlapping regions by means of a grid, each region being a portion of the grid around a grid node, and constructing curves representing the Taylor polynomials of degree k from the coefficients defining the temporary image at each grid node.

Fig. 16

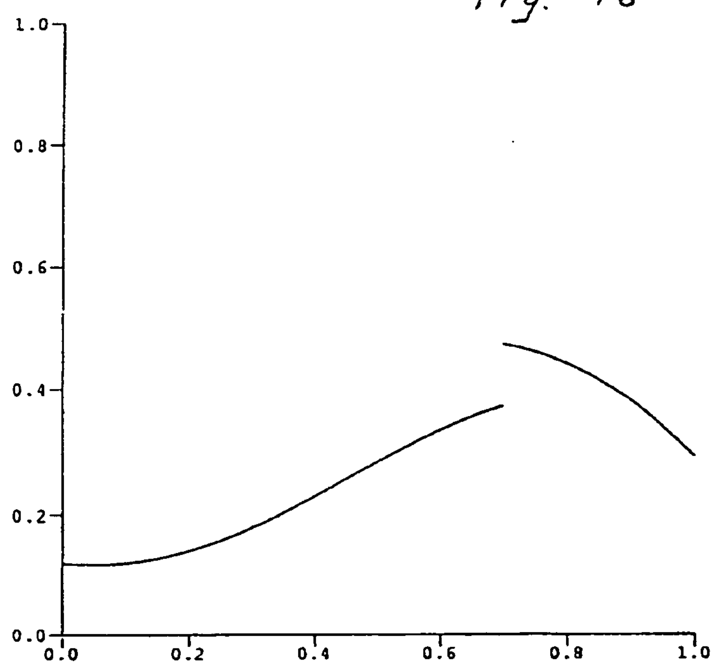


Fig. 2a

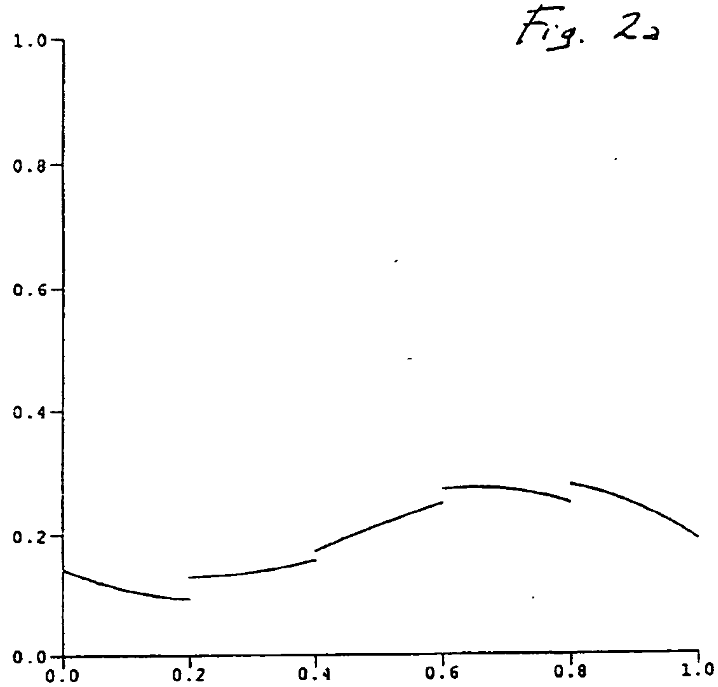


Fig. 26

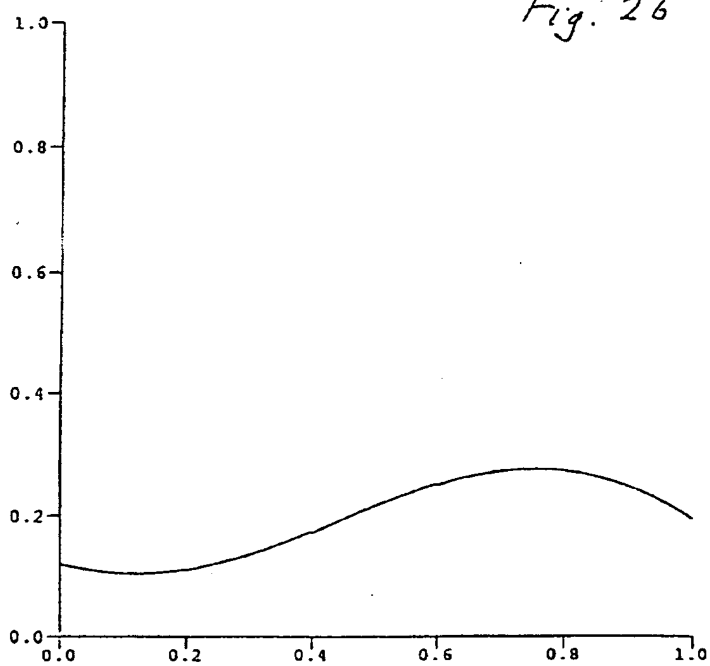


Fig. 3a

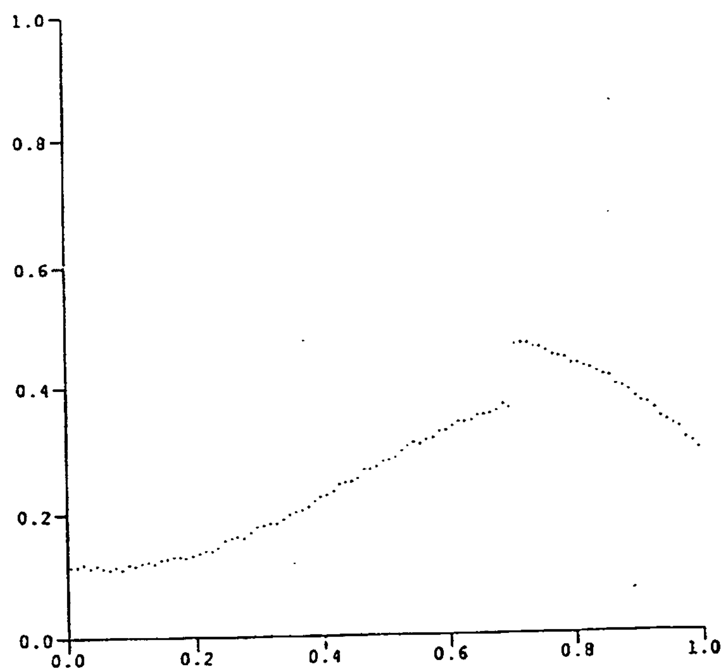
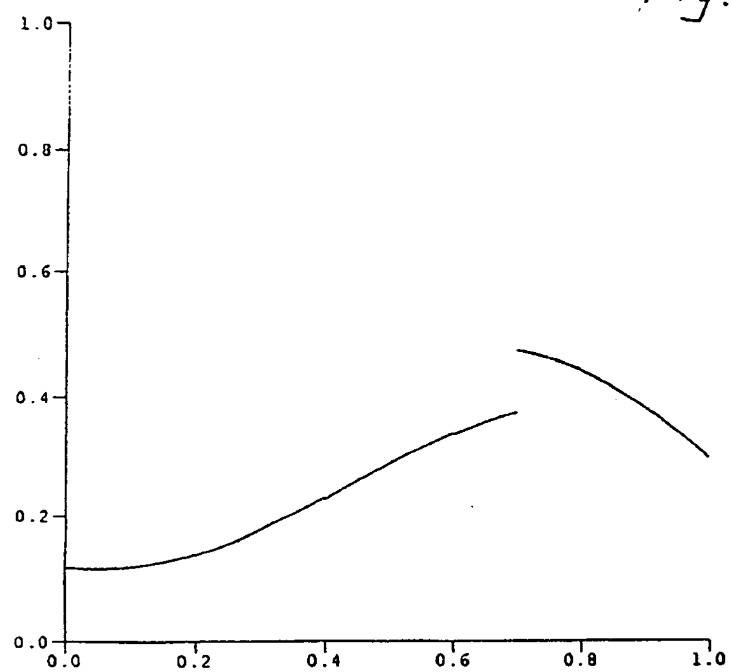


Fig. 36



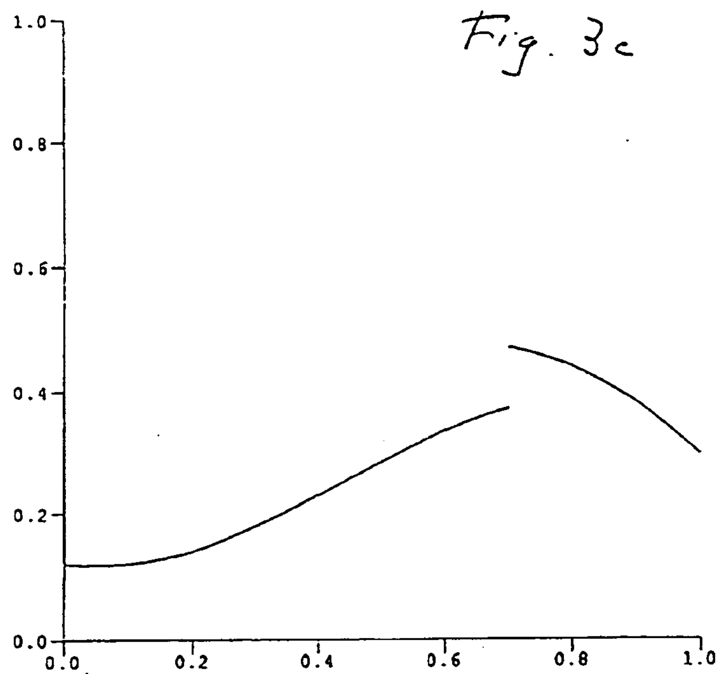


Fig. 3c

Fig. 3d

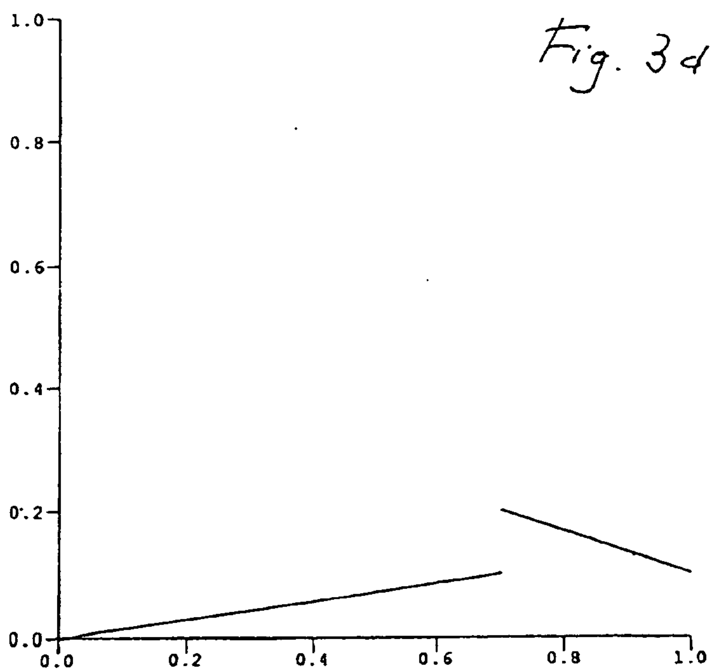


Fig. 4a

Fig. 4b

Fig. 5

Fig. 6a

Fig. 66

Fig. 16

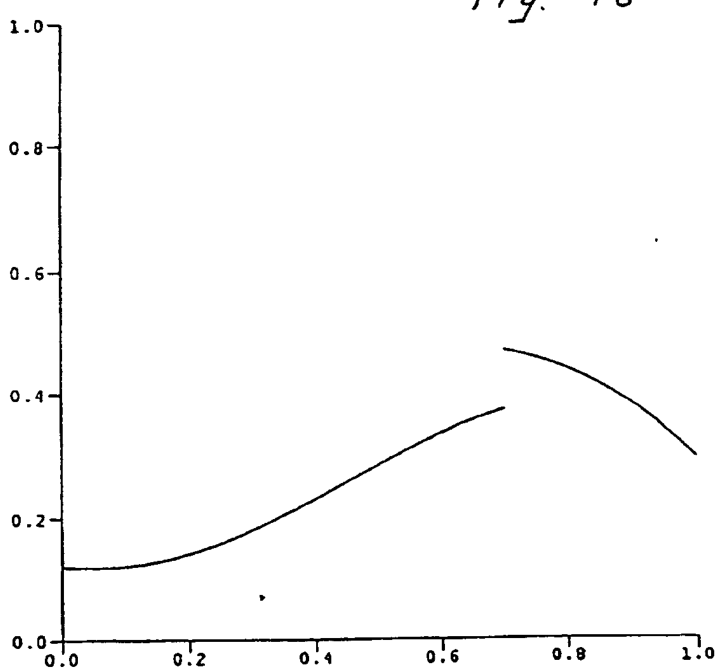
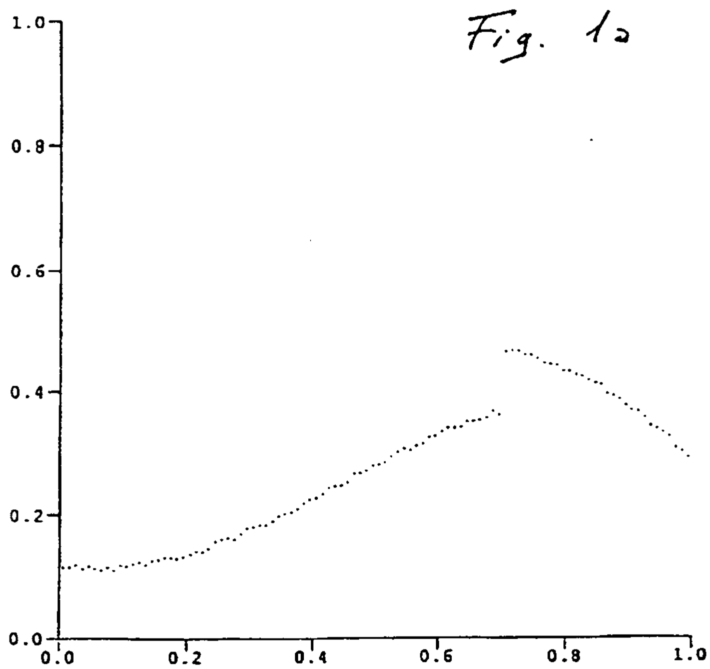


Fig. 1a



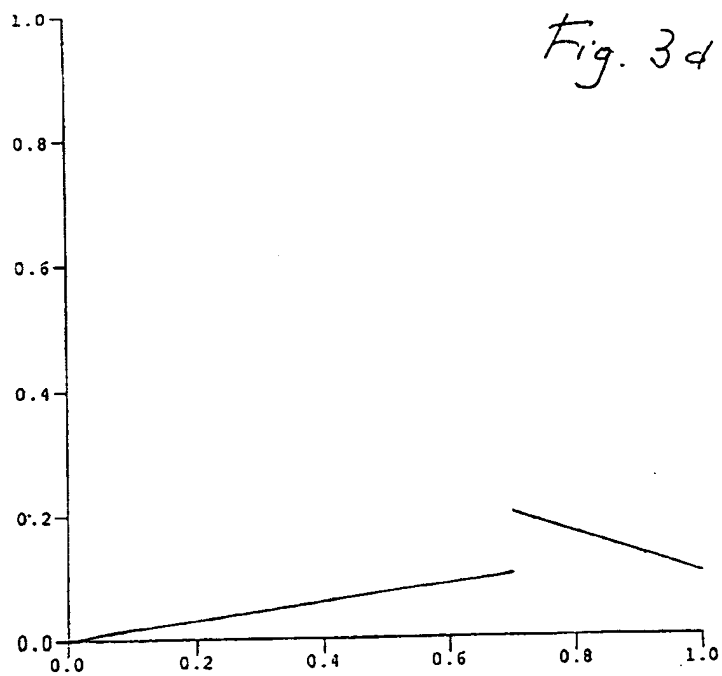


Fig. 3d

Fig. 2a

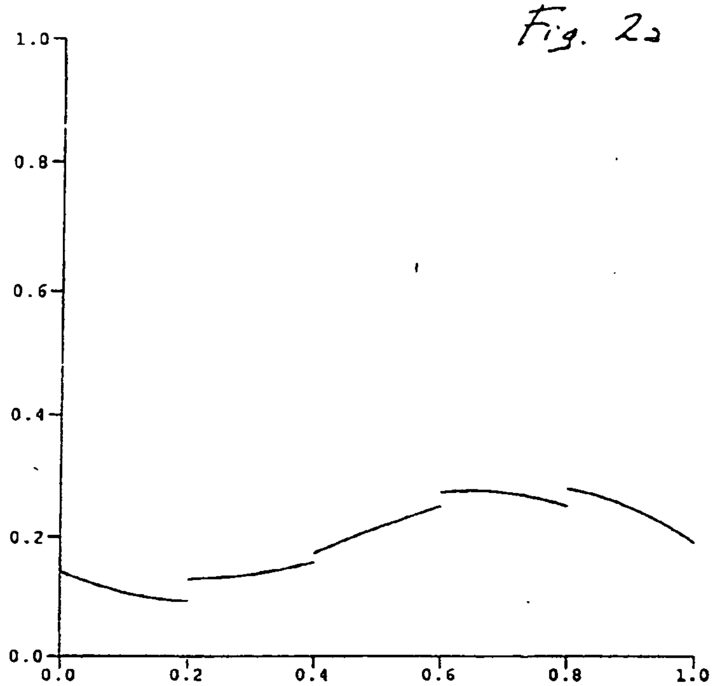


Fig. 3c

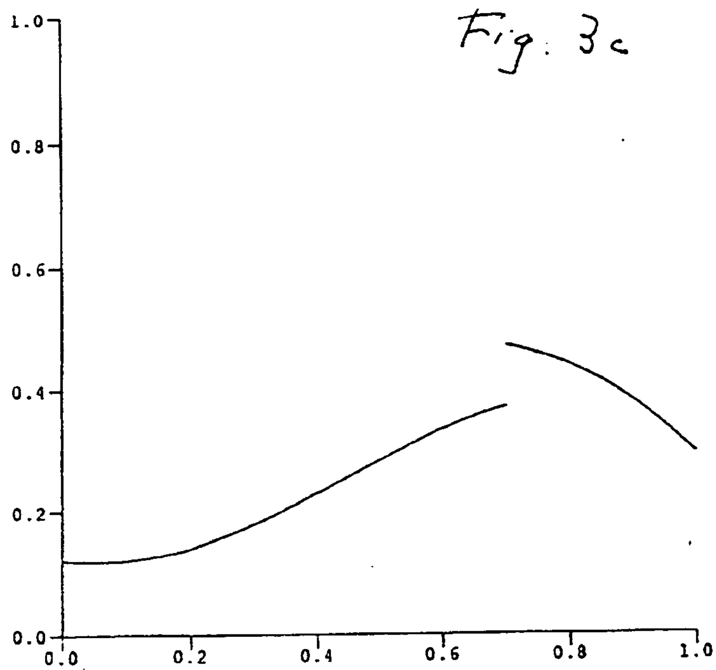


Fig. 4b

Fig. 4a

Fig. 5

Fig. 6a

Fig. 66